Game-Theoretical Pragmatics

Gerhard Jäger
Gerhard.Jaeger@uni-bielefeld.de

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1 Introduction

In his book *Using Language* (Clark 1996), Herb Clark makes a distinction between two approaches to the psychology of language. He favors what he calls an *action approach* to language use and distinguishes it from the more traditional *product approach*. The latter line of research focuses on linguistic *structures*, the former on *processes*. Clark's distinction is also useful when considering languages of logic and logic-based approaches to natural language. While the language-as-product perspective may represent the mainstream of the logic-and-language line of research, the procedural aspects of language have been stressed over and over again by many researchers. (It is not a coincidence that a rather influential book on logical grammar, van Benthem 1991, is titled "Language in Action".) This ranges from proof theoretic investigations into the algorithmic aspects of reasoning via the automata-theoretic reconstruction of generation and parsing of syntactic structures to the various brands of dynamic semantics that model the effects that the process of interpreting an expression has on the agent who does the interpreting.

It is only natural to move one step further, from language-as-action to *language-as-**inter**action*. To get the full picture, the social aspects of language as a medium for communication have to be taken into account. If we further assume that the interacting agents are able to choose between different possible actions, and that they have preferences regarding the outcome of their interaction, we are entering the realm of game theory.

The best worked-out implementation of such an approach is *Game-theoretic semantics* in the sense of Hintikka (1973). Here the process of interpreting a syntactically complex sentence S is modeled as a game. One can imagine the situation as a debate where one player, the *proponent*, tries to convince an invisible audience of the truth of S, while the *opponent* tries to disprove S. To take a simple example, consider the first-order sentence

$$\forall x \exists y (x = y) \tag{1}$$

As S starts with a universal quantifier, the opponent is free to choose some object from the universe of discourse as value for the bound variable. The subsequent existential quantifier means that the proponent can choose some object as value for y. After the players made their choices, the remaining atomic formula is evaluated. If it is true, the proponent wins, otherwise the opponent wins. In this particular game, it is obvious that the proponent has a winning strategy: always choose the same object as the opponent! Generally, if the proponent has a winning strategy for a particular model, the formula is true in this model.

Game-theoretic semantics offers an alternative to Tarskian semantics because the meaning of a complex formula is not computed in terms of the truth conditions of its components, but with reference to the sub-games that correspond to these components. The rules of the game can be modified in various ways regarding the pieces of information that the players have at their disposal at the various positions within the game tree. The most important application of this approach is perhaps the game-theoretic analysis of IF-logics ("Independence friendly logics"). To modify the previous example minimally, consider the following formula:

$$\forall x \exists y / \{x\} (x = y) \tag{2}$$

The slash notation indicates that the choice of a value for y has to be independent of the value of x. Technically speaking, the proponent now has *imperfect* information at the second stage of the game because she does not know which move the opponent made in the first stage. In this particular example, this implies that neither player has a winning strategy if the domain of discourse contains at least two element. So the formula does not have a definite truth value.

Varying the rules of the game along these lines leads to interesting analyses of concepts like branching quantifiers, but also of intricate natural language phenomena like donkey anaphora. The interested reader is referred to Hintikka and Sandu's article in this handbook (Hintikka and Sandu 1997) for an in-depth discussion. A more recent representative collection of papers about this version of game theoretic semantics (which also contains abundant material about linguistic applications of signaling games) can be found in Pietarinen (2007).

2 Signaling games

As was said above, Hintikka style semantic games can be conceptualized as zero-sum games between two players that try to win a debate (about the truth of the sentence in question) in front of an audience. This basic scenario has also been employed by several authors to model the act of *choosing an expression*. Here we enter the realm of communication and pragmatics. This language-as-a-debate model of pragmatics was worked out in some detail by Arthur Merin (see for instance Merin 1999a,b), building on earlier work by Ducrot (1973) and Anscombre and Ducrot (1983). In a series of recent publications, Jacob Glazer and Ariel Rubinstein develop a theory of the "pragmatics of debate". They also model communication as an argumentation game between two competing agents, trying to convince a third party (see for instance Glazer and Rubinstein 2004, 2005). Considering the issue from an economist's perspective, they are mainly interested in the issue how the rules of the debate affect the amount of useful information that the third party will extract from listening to such a competitive debate.

Besides this language-as-a-debate model, there is another tradition of game theoretic models of communication that goes back to the work of Lewis (1969). Here, communication is essentially seen as an attempt of a sender to manipulate a receiver by transmitting certain signals that might influence future decisions of the receiver. If we assume that the interests of the players are common knowledge, such a manipulation will only be successful if the game is not zero-sum. (In a purely competitive game like poker, rational players are well-advised **not** to give away any private information by sending signals!) So in this tradition, communication is essentially seen as part of an at least partially collaborative endeavor.

The simplest model of this kind are *signaling games* that were originally proposed by Lewis in his dissertation, and later refined (and repeatedly reinvented) by economists and biologists.

In a signaling game we have two players, the sender (let us call her Sally) and the receiver (whom I will call Robin). Sally has some private information that Robin lacks. In the first stage of the game, Sally sends a message to Robin. The choice of the message may depend on Sally's private information. Robin in turn chooses an action, possibly dependent on the message that he observed. Both players have a preference ordering over possible (message,action) sequences that is captured by utility functions.

A strategy for Sally in such a game is a function from information states to messages, while a strategy for Robin would be a function from messages to actions. Lewis focuses on the *Nash-equilibria* on such games. A Nash equilibrium is a configuration of strategies that is self-reinforcing in the sense that no player has an incentive to deviate from it provided he has reason to believe that everybody else is abiding it. For instance, if it is generally believed that nodding means affirmation and shaking the head means negation, then the best thing to do is to nod if you want to express "yes", and to shake your head if you want to express "no". This is an equilibrium that works fine in western Europe. However, the opposite convention is also an equilibrium, as can be observed in parts of south-east Europe. There is nothing intrinsically affirmative or negative in nodding or shaking one's head.

Lewis' main point was to demonstrate that signals may be associated with a meaning in an arbitrary way without reference to a prior negotiation in some meta-language (an assumption which would lead to an infinite regress). Rational players in a signaling game use/interpret signals in a certain way because they have certain expectations about the behavioral dispositions of the other players, and it doesn't matter how these expectations are justified.

Lewis' solution is arguably incomplete because it does not solve the problem of *equilibrium selection*. As the previous example illustrates, there may be several Nash equilibria in a game (affirmative nodding and negative head-shaking; affirmative head-shaking and negative nodding), and the players have no *a priori* reason to favor one over the other. The best candidate for a causal explanation here is precedent. But this seems to lead to an infinite regress again.

The evolutionary properties of signaling games have been studied extensively in the context of Evolutionary Game Theory. The results from this strand of research go a long way to actually solve Lewis' problem. In an evolutionary setting, a game is not played just once but many times over and over again. There is some positive feedback from the utility that a certain strategy σ achieves on average, and the likelihood with which σ will be played in future iterations. If the interests of sender and receiver are sufficiently aligned—i.e. they both have an interest in successful communication—almost all initial states will evolve into a final state where signals do carry a meaning about the information state of the sender. The phrase "almost all initial states" is to be interpreted in its measure-theoretic sense here; even though there are initial states that do not evolve into meaningful signaling systems, their probability is infinitesimally small. Whether or not evolution leads to an optimal communication system depends on specific details of the underlying evolutionary dynamics. A discussion would lead beyond the scope of this article. As a rule of thumb, a deterministic dynamics may get stuck in sub-optimal states, while a small but non-negligible amount of random noise favors the emergence of optimal signaling systems.

In the biological domain, the positive reinforcement mentioned above is implemented via increased biological fitness as the result of successful communication. The logic of evolution, however, also applies if reinforcement is mediated via imitation and learning. So the emergence of stable equilibria in iterated signaling games can also be explained via cultural evolution.

The evolutionary stability of various classes of signaling systems were established, inter alia, in Blume et al. (1993), Wärneryd (1993) and Trapa and Nowak (2000). Huttegger (2007), Pawlowitsch (2008) and Jäger (2008a) explore under what conditions a system will converge towards a sub-optimal state with a positive probability. In van Rooij (2004) and Jäger (2007) the hypothesis is entertained that natural languages constitute equilibria of signaling games, and that therefore high-likelihood equilibria correspond to recurrent patterns in the languages of the world.

To conclude this section about signaling games and Nash equilibria, I'd like to mention an interesting connection of this kind of games to Hintikka style semantic games for IF-logics. Sandu and Sevenster (2008) point out that Hintikka games can be reconceptualized in a way that they contain signaling games as sub-games. Consider the following IF formula:

$$\forall x \exists y \exists z/\{x\}(x = z) \tag{3}$$

Even though this looks similar to the formula in (2), here the proponent has a winning strategy for domains with multiple elements. He can choose y to be identical to x, and z to be identical to y. You can consider such a game as a game between two coalitions of players, the proponents and the opponents. Every member of a coalition is responsible for exactly one variable. In the game corresponding to (3), the y-player knows which value the opponents chose for x, and he wants to communicate this to the z-player. Choosing the value of y to be identical to x is a way to signal this information to z. So here the winning strategy contains an equilibrium of an embedded signaling game.

3 Rational communication

3.1 Equilibrium analyses

The insight that signaling games are being played iteratively by the members of a population helps to explain how signals may acquire and sustain a conventional meaning in the first place. However, even if a certain meaning-signal association is part of an equilibrium in the long run, it might not be rational to play that equilibrium in one particular situation. For instance, if a driver asks a passerby:

(1) Is this the right way to the station?

shaking the head (in western Europe) may be a semantically correct response, but a cooperative speaker will augment this with more detailed instructions how to get to the station. In other words, even if a set of semantic conventions are common knowledge, it is possible that some of them are not rational in a particular situation. It might even be rational to diverge from the conventions. This fact is well-known since the work of Grice (1975), and it has also frequently been observed that Gricean pragmatics has a strong game-theoretic flavor because it involves strategic rationality considerations.

In a series of publications (see for instance Parikh 1987, 1991, 2001), Prashant Parikh has developed a generalization of Lewisian signaling games that model the inference from an exogenously given semantic convention to the actual pragmatic interpretation of a signal. His model differs from Lewis' in a small but crucial way. In standard signaling games, Sally has the same set of messages at her disposal in each situation. Also, Robin has a certain set of actions at his disposal which does not depend on the message that he receives.

In Parikh style games,[1] the set of messages that Sally can send may differ from situation to situation, and the set of actions that Robin may take may differ from message to message. More specifically, Robin's possible actions are identified with the possible readings of the message that he received. So the literal meaning (or meanings, in the case of ambiguous messages) of a message is part of the structure of the game itself. Also, in a given situation Sally can only send messages that are true in this situation according to their literal interpretation. So the truth conditions of messages are also part of the structure of the game.

Parikh assumes that rational agents will settle on a Nash equilibrium of such a game, and only use messages that conform to this equilibrium. To see this from Robin's perspective, the very structure of the game ensures that he can infer the truth of a message (in one of its readings) from the fact that Sally has sent it. If he has furthermore reason to believe that Sally plays according to a certain equilibrium, he can infer from an observed message that Sally is in a state where this message belongs to her equilibrium strategy. The former kind of inference is based on the meaning of the message; the latter is based on the belief in the sender's rationality. Therefore the latter can aptly be called a pragmatic inference or an implicature.

[1] To stress this difference, Parikh speaks of *games of partial information* rather than signaling games.

Parikh style games may have more than one Nash equilibrium. Parikh therefore ponders criteria for equilibrium selection, and he argues in favor of the notion of a Pareto-Nash equilibrium as the appropriate concept.[2]

It is a consequence of Parikh's model that the sender always utters a message that is true according to its literal meaning in one of its readings. This does not only exclude lying but also any kind of non-literal interpretation like hyperboles or metaphors. Consequently, his model is successful mainly for studying the pragmatic resolution of underspecification, like the resolution of ambiguities or the computation of scalar implicatures.

Other approaches maintain the standard signaling game assumption that the set of feasible messages does not depend on the private information of the sender. Instead, the association between information states and messages that is defined by the literal meaning of messages is given a special status in the reasoning of the agents. Also, it is not so clear whether the solution concept of a Nash equilibrium (or strengthenings thereof) is really appropriate to model the action of rational agents in one-shot games.

3.2 The Iterated-Best-Response model

Nash equilibria are self-reinforcing ways to play a game. If every player has reason to believe that the other players play according to a certain equilibrium, it is rational to stick to play that very equilibrium oneself. However, it is not immediately obvious what these reasons would be. This question is especially pressing if we are dealing with one-shot games where precedent or evolutionary arguments cannot be used. Signaling games usually have many equilibria.[3] So the requirement that every player plays according to some equilibrium strategy is not much stronger than the requirement of rationality (plus common knowledge that each player is rational). Therefore refinements of the equilibrium concept (like Parikh's proposal to focus on Pareto-Nash equilibria) are necessary to derive non-trivial prediction. But even such refinements often do not guarantee a unique solution.

On the other hand, equilibrium analyses place a considerable burden on the cognitive capabilities of the players. Calculating the set of equilibria of a sequential game is a complex task. Research in behavioral game theory has shown that actual humans, in experimental settings, usually do not play according to some Nash equilibrium. Rather, test persons employ step-by-step reasoning heuristics when making a decision in a strategic situation (see for instance Selten 1998).

For these reasons, various authors (like, *inter alia*, Rabin 1990; Benz and van Rooij 2007; Franke 2008a,b; Jäger 2008b) have explored solution concepts that do not make use of the notion of an equilibrium. Instead they have suggested iterative reasoning protocols to describe the rational usage of messages with an exogenously given meaning in a given context. The basic idea that is common to these approaches can roughly be described as follows: One player, A, starts their reasoning process with the provisional assumption

[2] Parikh calls an equilibrium a Pareto-Nash equilibrium if it is not possible to switch to another equilibrium in such a way that all players receive a higher utility in the new equilibrium.

[3] See for instance the discussion in Battigalli (2006).

that the other player, B, follows the semantic convention. Based on this assumption, B chooses the best possible strategy, i.e. the strategy that maximizes B's payoff. If the semantic convention happens to be identical to a Nash equilibrium, the story ends here. Otherwise, A might anticipate B's reasoning step and revise their decisions accordingly. This procedure may be arbitrarily iterated many times.

There are many technical differences between the mentioned approaches, relating to the questions which player starts the reasoning procedure, how many iterations are considered, how are multiple best responses to a certain assumption reconciled etc. In the sequel I will present one particular implementation in finer detail.

3.2.1 Rationalizability, strong belief and justifiable decisions

Let us start with a simple example. Suppose Sally is in either of two states. (For the time being, we assume that all relevant aspects of the interaction are determined in each state, so we can equate them with possible worlds.) She either prefers tea (w_1) or she prefers coffee (w_2). Robin can take either of two actions: he can serve her tea (a_1) or coffee (a_2). However, Robin does not know in which state Sally is. He considers both worlds as equally likely. Both Sally and Robin prefer a scenario where Sally gets her favorite beverage. These preferences are captured by a two-place function from worlds and actions to real numbers $v_k \in \mathbb{R}^{W \times A}$ for each player $k \in \{S, R\}$. If v is finite, it can be represented by a utility matrix where rows represent possible worlds and columns represent actions. The two numbers in each cell give Sally's and Robin's utilities respectively. Before Robin takes

	a_1	a_2
w_1	$1; 1$	$0; 0$
w_2	$0; 0$	$1; 1$

Table 1: A simple coordination scenario

an action, Sally can send one out of two messages, m_1 ("I prefer tea.") or m_2 ("I prefer coffee."). The literal interpretation of the messages is common knowledge: $\|m_1\| = \{w_1\}$ and $\|m_2\| = \{w_2\}$.

A strategy for Sally is a function which determines for each world which message she sends. Likewise, a strategy for Robin is a function from messages to actions. The set of sender strategies is denoted by $\mathcal{S} = M^W$, and the set of receiver strategies by $\mathcal{R} = A^M$. Which strategies will Sally and Robin choose if all that is common knowledge is the structure of the game, the literal meaning of the messages, and the fact that they are both rational?

To address this question, we need some notation. The *utility functions* for the players determine their payoff for a particular instance of the game, i.e. a particular world, a message, and an action. Let W be the set of possible worlds, M the set of signals and A

the set of actions. Then both u_s and u_r (Sally's and Robin's utility functions respectively) are functions from $W \times M \times A$ into $\mathbb{R}$.

For the time being we assume that "talk is cheap"—the utility depends only on v_k, not on the message send:

$$u_k(w, m, a) = v_k(w, a)$$

Let p^* be a probability distribution over W, i.e. $p^* \in \Delta(W)$,[4] such that for all w : $p^*(w) > 0$. It represents Robin's prior assumptions on the probability of the possible worlds.

The expected utility of a player $k \in \{S, R\}$ for a pair of strategies (s, r) is given by

$$u_k(s, r) = \sum_{w \in W} p^*(w) u_k(w, s(w), r(s(w)))$$

(Note the the symbols u_s and u_r are overloaded here, referring both the utility functions and the expected utilities. No confusion should arise from this though.)

On the basis of the publicly available information, Sally will figure out that the set of strategies that Robin could possibly play if it is common knowledge that both players are rational is some set $\mathbf{R}$. So Sally's expectation about Robin's strategy can be represented as a probability distribution P over $\mathbf{R}$. If Sally is rational, she will play a strategy that maximizes her expected utility, given P. Since Robin does not know P, all he can figure out is that Sally will play some strategy that maximizes her expected utility for some P. The same kind of argument can be made with respect to Robin's considerations. Let us make this more precise.

Even if Sally does not know for sure which strategy Robin plays, she has some expectations about Robin's possible reactions to each of the messages. Formally, Sally's first order beliefs[5] about Robin are captured by a function ρ that maps each message to a probability distribution over actions. We write $\rho(a|m)$ for the probability that ρ assigns to action a if Sally sends message m. Likewise, Robin's first order beliefs are captured by a function σ_1 from worlds to probability distributions over messages. From this he can derive his posterior beliefs σ_2, which is a function from messages to probability distributions over worlds.

A rational player will always play a strategy that maximizes his expected utility, given his beliefs. The notion of a *best response* captures this.

Definition 1 (Best response to beliefs) *Let $\sigma_2 \in \Delta(W)^M$ be a posterior belief of the receiver, and $\rho \in \Delta(A)^M$ be a first order belief of the sender. The sets of* best responses

[4] I use the convention whereas for each finite set X,

$$\Delta(X) = \{p \in [0, 1]^X | \sum_{x \in X} p(x) = 1\}$$

is the set of probability distributions over X.

[5] First order beliefs are beliefs that only concern the actions of the other player. Second order beliefs would also include assumptions about the first order beliefs of the other player, etc.

to these beliefs are defined as follows:

$$\begin{aligned} BR_s(\sigma_2) &= \{r \in \mathcal{R} | \forall m. r(m) \in \arg\max_{a\in A} \sum_{w\in W} \sigma_2(w|m) u_r(w,m,a)\} \\ BR_r(\rho) &= \{s \in \mathcal{S} | \forall w. s(w) \in \arg\max_{m\in M} \sum_{a\in A} \rho(a|m) u_s(w,m,a)\} \end{aligned}$$

The posterior beliefs σ_2 can usually be derived from Robin's first order beliefs and his prior belief p^* by using Bayesian updating:

$$\begin{aligned} \sigma_2(w|m) &= \frac{\sigma_1(m|w)p^*(w)}{\sum_{w'\in W} \sigma_1(m|w')p^*(w')} \\ \text{provided} \quad & \max_{w'\in W} \sigma_1(m|w') > 0 \end{aligned} \tag{4}$$

If Robin encounters a message that had probability 0 according to his first order beliefs, he has to revise those beliefs. Different belief revision policies correspond to different restrictions on the formation of posterior beliefs.

Battigalli and Siniscalchi (2002) propose the notion of *strong belief*. An agent strongly believes a certain proposition A if he maintains the assumption that A is true even if he has to revise his beliefs, provided the new evidence is consistent with A. Now suppose that Robin strongly believes that Sally plays a strategy from the set $\mathbf{S}$. Then the formation of the posterior belief is subject to the following constraint.

Definition 2 *Let $\mathbf{S} \subseteq \mathcal{S}$ be a set of sender strategies and σ_1 be a first order belief of the receiver. σ_2 is a possible posterior belief for σ_1 and $\mathbf{S}$ ($\sigma_2 \in \text{posterior}(\sigma_1, \mathbf{S})$) if the following conditions are met:*

1. $\sigma_2 \in \Delta(W)^M$
2. *If* $\max_{w\in W} \sigma_1(m|w) > 0$, *then*
$$\sigma_2(w|m) = \frac{\sigma_1(m|w)p^*(w)}{\sum_{w'\in W} \sigma_1(m|w')p^*(w')}.$$
3. *If* $\max_{w\in W} \sigma_1(m|w) = 0$ *and* $m \in \bigcup_{s\in\mathbf{S}} \text{range}(s)$, *then there is some probability distribution* $P \in \Delta(\mathbf{S})$ *such that* $m \in \bigcup_{s\in\text{support}(P)} \text{range}(s)$ *and some prior belief* $\sigma_1' \in \Delta(M)^W$ *with* $\sigma_1'(m|w) = \sum_{s:s(w)=m} P(s)$ *for all* w *and* m, *such that*
$$\sigma_2(w|m) = \frac{\sigma_1'(m|w)p^*(w)}{\sum_{w'\in W} \sigma_1'(m|w')p^*(w')}.$$

The second clause captures the case where the observed message has a positive prior probability, and the posterior belief can be derived via Bayesian updating. The third clause captures the case where the observed message m had a prior probability of 0, but is consistent with the assumption that Sally plays $\mathbf{S}$. In this case the prior probability is revised to some alternative prior σ_1' that is consistent with the belief that Sally plays strategies from $\mathbf{S}$. The posterior belief is then formed by applying Bayes' rule to σ_1'. If the observed message is inconsistent with the belief that Sally plays a strategy from $\mathbf{S}$, no restriction on the belief revision policy are imposed.

Now let X be a set of strategies of some player. The best responses to X are the set of strategies that the other player might conceivably play if he is rational and he strongly believes that the first player plays X.

Definition 3 (Best response to a set of strategies) *Let* $\mathbf{S} \subseteq \mathcal{S}$ *and* $\mathbf{R} \subseteq \mathcal{R}$ *be sets of strategies.*

$$\begin{aligned}
BR_r(\mathbf{S}) &= \{r \in \mathcal{R} | \exists P \in \Delta\mathbf{S} \exists \sigma_1 (\forall m, w. \sigma_1(m|w) = \sum_{s \in \mathbf{S}: s(w)=m} P(s)) \\
&\quad \exists \sigma_2 \in \text{posterior}(\sigma_1, \mathbf{S}) : r \in BR_r(\sigma_2)\} \\
BR_s(\mathbf{R}) &= \{s \in \mathcal{S} | \exists P \in \Delta(\mathbf{R}) \exists \rho (\forall a, m. \rho(a|m) = \sum_{r \in \mathbf{R}: r(m)=a} P(r)) : \\
&\quad r \in BR_s(\rho)\}
\end{aligned}$$

Suppose a player has figured out, by just using publicly available information, that the other player will play a strategy from the set X. If the player is rational, he will play a best response to X. Let Y be the set of best responses to X. The other player is perfectly able to come to the same conclusion. He will thus play any of the best responses to Y. If the considerations that led the first player to the assumption that the second player uses a strategy from X were correct, the set of best responses to Y should equal X, and vice versa. This is exactly the intuition that is captured by the notion of a *strongly rationalizable equilibrium* (SRE).[6]

Definition 4 (Strongly rationalizable equilibrium) $(\mathbf{S}, \mathbf{R}) \in POW(\mathcal{S}) \times POW(\mathcal{R})$ *is a* strongly rationalizable equilibrium *iff*

$$\begin{aligned}
\mathbf{S} &= BR_s(\mathbf{R}) \\
\mathbf{R} &= BR_r(\mathbf{S})
\end{aligned}$$

A game may have more than one of such equilibria though. Let us consider our example again. The four sender strategies can be denoted by $11, 12, 21$ and 22, where the first and the second digit give the index of the message that Sally sends in world w_1 and w_2 respectively. Robin's strategies can be coded the same way.

There are three SREs for this game:

[6]The notion of a *rationalizable equilibrium* is related to Bernheim's (1984) and Pearce's (1984) notion of *rationalizability.* Because we demand strong belief rather than simple belief, not every rationalizable strategy is part of some strongly rationalizable equilibrium. Also note that Stalnaker (1997) uses the notion of "strongly rationalizable equilibrium" is a different sense.

1. $(\{12\}, \{12\})$
2. $(\{21\}, \{21\})$
3. $(\{11, 12, 21, 22\}, \{11, 12, 21, 22\})$

The first one seems reasonable—Sally always sends an honest signal, and Robin believes her. In this way, both obtain the maximally possible utility. The second equilibrium is the one where Sally uses the messages ironically, and Robin is aware of that. It also yields the maximal payoff. However, to coordinate on this equilibrium, the players ought to have some clue that Sally is ironic. If they have no *a priori* information about each other, this equilibrium is counter-intuitive.

The third equilibrium is the one where the information about the literal interpretation of the messages plays no role. If only rationality considerations are taken into account, every strategy can be justified. The expected utility of the players could be 1, but also just 0 or 0.5.

The criterion that we employed here to single out the first equilibrium seems to be something like *Choose the equilibrium where Sally always sends a true message!* However, there may be equilibria where honesty is not rational. Consider the example given in Table 2, taken from Rabin (1990). All three worlds are assumed to be equally likely according to p^*.

	a_1	a_2	a_3
w_1	$10; 10$	$0; 0$	$0; 0$
w_2	$0; 0$	$10; 10$	$5; 7$
w_3	$0; 0$	$10; 0$	$5; 7$

Table 2: Partially aligned interests

As far as Robin is concerned, the best action in world w_k is a_k, for all indices $i \in \{1, 2, 3\}$. However, Sally would prefer Robin to perform a_2 both in w_2 and in w_3. So while Sally would prefer Robin to know the truth in w_1 and in w_2, she has an incentive to make Robin believe that w_2 is the case if she is actually in w_3. Robin is of course aware of this fact. So if it is common knowledge that both players are rational, they will not agree on a communication system that reliably distinguishes between w_2 and w_3. Both do have an incentive though to distinguish $\{w_1\}$ from $\{w_2, w_3\}$.

Let us assume that there are three messages, m_1, m_2, and m_3, with the literal interpretations $\|m_k\| = \{w_k\}$ for all i. Strategies are denoted by triples of similarly to the previous example.

The honest sender strategy, call it h, is 123. However, $BR_r(\{123\}) = \{123\}$, and $BR_s(\{123\}) = \{122\}$. So unlike in the previous example, $(\{h\}, BR_r(\{h\}))$ does not form a SRE. Still, it is possible to make intuitively plausible predictions here.

The essential idea is that each player should be able to justify his choice of a strategy. One possible justification for a sender strategy is honesty. Another possible justification is rationality: a given strategy is justifiable if it is a best response to a set of justifiable strategies of the other player. Nothing else is justifiable.

So formally we can define the set of justifiable strategies as the smallest pair of sets $J = (J_s, J_r)$ with

$$\begin{aligned} h &\in J_s \\ BR_s(J_r) &\subseteq J_s \\ BR_r(J_s) &\subseteq J_r \end{aligned}$$

Since J is defined as a smallest fixed point of a monotonic[7] operator, we can equivalently define it cumulatively as

$$\begin{aligned} H_0 &= \{h\} \\ H_{n+1} &= H_n \cup BR_s(BR_r(H_n)) \\ H_\omega &= \bigcup_{n\in\mathbb{N}} H_n \\ J &= (H_\omega, BR_r(H_\omega)) \end{aligned}$$

In Rabin's example, we have

$$J = (\{122, 123, 132, 133\}, \{122, 123, 132, 133\}).$$

This set forms an SRE. So we can infer that Sally will always send message m_1 in w_1, and she will never use that message in another world. Robin will always react with a_1 to m_1. No further predictions can be derived.

In this example, J is an SRE. This is not necessarily the case. In the next example, there are two worlds, two messages, and three actions. The utility function is given in Table 3.

	a_1	a_2	a_3
w_1	$10;10$	$0;0$	$1;7$
w_2	$10;0$	$0;10$	$1;7$

Table 3: Another example with partially aligned interests

J is calculated in the following way:

$$\begin{aligned} H_0 &= \{12\} & BR_r(H_0) &= \{12\} \\ H_1 &= \{11, 12\} & BR_r(H_1) &= \{32, 12\} \\ H_2 &= H_1 \end{aligned}$$

[7] The BR_r-operator is not generally monotonic due to the belief revision policy for unexpected messages that is implicit in the definition of BR_r. It is monotonic though on a collection $S \subseteq POW(\mathcal{S})$ if for all $\mathbf{S} \in S : \bigcup_{s\in\mathbf{S}} \text{range}(s) = M$.

So $J = (\{12, 11\}, \{12, 32\})$. However, J is not an SRE, because h is not a best response to J_r. So even though h can be justified because it is the honest strategy, it is never rational to use it because it is not a best response to the set of justifiable receiver strategies. Thus a rational player wouldn't use it and this undermines the justification for the other strategies from J.

This example illustrates that honesty is not a sufficient justification for some sender strategy. Rather, h must be doubly justifiable—a rational player might use it (a) because its the honest strategy, and (b) because rationality considerations do not speak against using it. This is the case if and only if J is a strongly rationalizable equilibrium.

The discussion so far can be summarized by the following principle, which rational interlocutors are assumed to obey:

> **Pragmatic rationalizability:** If J is a strongly rationalizable equilibrium, pragmatically rational players will play strategies from J.

If J is an SRE, we call the strategies from J the *pragmatically rationalizable strategies*.

4 Information states and message costs

In the previous section, we assumed that Sally has complete knowledge about the state of the world. This is quite unrealistic; Sally might have only partial knowledge herself. She is in a certain information state, which is a non-empty subset of W. The set of information states is called I.

There is some function ξ that maps each world w to a probability distribution $\xi(\cdot|w)$ over I, such that

$$\sum_{i \ni w} \xi(i|w) = 1$$

In each world w, nature assigns the information state $i \ni w$ to Sally with probability $\xi(i|w)$.[8]

In the refined model, strategies for Sally are functions from I to M. If Sally is in an information state $i \subseteq W$, her private beliefs contain some probability distribution over W. This distribution, call it $P(\cdot|i)$, is given by

$$P(w|i) = \frac{\xi(i|w)p^*(w)}{\sum_{w' \in W} p^*(w')\xi(i|w')}.$$

Robin's epistemic state encompasses a probability distribution $q^* \in \Delta(I)$, his subjective probabilities that Sally is in a particular information state. It is given by

$$q^*(i) = \sum_{w} p^*(w)\xi(i|w).$$

[8] I assume for the time being that Sally never has incorrect information.

As a further refinement, we assume sending a message may incur a cost for Sally, and that these costs may differ between messages (regardless of the state Sally is in, and of Robin's interpretation of the message). Formally, there is some function $c \in \mathbb{R}^M$ that assigns costs to messages, and these costs are subtracted from Sally's utility. The relative weight of the payoff that is due to Robin's action and the costs that sending a message incurs need not be common knowledge. Rather, there is a set of cost functions C, and Sally makes a private assessment about the probabilities of these cost functions. So Sally's private information state consists of the function ρ that defines a probability distribution over actions for each message, and a probability distribution $\gamma \in \Delta(C)$ over cost functions.[9]

The expected utility in a certain information state i, depending on some γ, is given by

$$\begin{aligned}
u_s(i, m, a; \gamma) &= \sum_{w \in i} P(w|i) \sum_{c \in C} \gamma(c)(v_s(w, a) - c(m)) \\
u_r(i, m, a; \gamma) &= \sum_{w \in i} P(w|i) v_r(w, a).
\end{aligned}$$

The normalized expected utilities for a strategy pair are

$$u_k(s, r; \gamma) = \sum_{i \in I} q^*(i) u_k(i, s(i), r(s(i)); \gamma).$$

The definitions from the previous section apply to the new model as well, with the modifications that

- possible worlds are to be replaced by information states,
- p^* is to be replaced by q^*, and
- the best response function has to be relativized to γ.

Here are the revised definitions:

Definition 5 (Best response to beliefs (revised version)) *Let $\sigma_2 \in \Delta(W)^M$ be a posterior belief of the receiver, $\rho \in \Delta(A)^M$ a first order belief of the sender, and γ a probability distribution over cost functions. The sets of* best responses *to these beliefs are defined as follows:*

$$\begin{aligned}
BR_s(\sigma_2, \gamma) &= \{r \in \mathcal{R} | \forall m. r(m) \in \arg\max_{a \in A} \sum_{w \in W} \sigma_2(w|m) u_r(w, m, a; \gamma)\} \\
BR_r(\rho, \gamma) &= \{s \in \mathcal{S} | \forall w. s(w) \in \arg\max_{m \in M} \sum_{a \in A} \rho(a|m) u_s(w, m, a, \gamma)\}
\end{aligned}$$

[9]This can be implemented within the standard framework of an extensive two-person game with incomplete information if we assume that there are different *types* of receivers that differ with respect to the sender's, but not with respect to the receiver's utility function.

Definition 6 (Best response to a set of strategies (revised version)) *Let* $\mathbf{S} \subseteq \mathcal{S}$ *and* $\mathbf{R} \subseteq \mathcal{R}$ *be sets of strategies.*

$$\begin{aligned}
BR_r(\mathbf{S}) &= \{r \in \mathcal{R} | \exists P \in \Delta\mathbf{S} \exists \sigma_1(\sigma_1(m|w) = \sum_{s \in \mathbf{S}: s(w)=m} P(s)) \\
&\quad \exists \sigma_2 \in \text{posterior}(\sigma_1, \mathbf{S}) \exists \gamma \in \Delta(C) : r \in BR_r(\sigma_2, \gamma)\} \\
BR_s(\mathbf{R}) &= \{s \in \mathcal{S} | \exists P \in \Delta\mathbf{R} \exists \rho(\rho(a|m) = \sum_{r \in \mathbf{R}: r(m)=a} P(r)) \\
&\quad \exists \gamma : r \in BR_s(\rho, \gamma)\}
\end{aligned}$$

Strategy s is an *honest* sender strategy if in each information state i, Sally chooses a message which denotes precisely her information state, and which is the cheapest such message according to some γ.

$$h = \{s | \exists \gamma \in \Delta(C) \forall i. s(i) \in \arg \min_{m: \|m\|=i} \sum_{c \in C} \gamma(c) c(m)\} \tag{5}$$

We make the *rich language assumption:* For each information state i, there is at least one message m with $\|m\| = i$. In this way it is guaranteed that an honest and rational strategy always exists.

Let us turn to an example which illustrates the computation of a scalar implicature. In a context where question (2a) is under discussion, the sentence (2b) will be pragmatically strengthened to the interpretation corresponding to sentence (2d). This is due to the fact that (b) and (c) form part of a scale, and if the speaker utters (b) and is known to be competent, her decision to utter (b) rather than (c) indicates that she considers (c) to be false.

(2)
a. Who came to the party?
b. Some boys came to the party.
c. All boys came to the party.
d. Some but not all boys came to the party.

To model this in a game, let us assume that there are two worlds, w_1 (where all boys came to the party) and w_2 (where some but not all boys came). Both Sally and Robin have an interest that as much information is conveyed to Robin as possible. There are three messages, m_1=(2c), m_2=(2d) and m_3=(2b). m_1 and m_3 are about equally complex, while m_2 is more complex. Let us say that there are two cost functions, c_1 and c_2, with $c_1(m_1) = c_1(m_3) = 0$ and $c_1(m_2) = 4$, and $c_2(m_k) = 0$ for all k.

Regarding the literal meanings of the messages, we have $\|m_1\| = \{w_1\}, \|m_2\| = \{w_2\}$, and $\|m_3\| = \{w_1, w_2\}$.

Robin has the choices to opt for w_1 (action a_1), for w_2 (action a_2) or to remain undecided and wait for further information (action a_3). The functions $v_{s/r}$ are given in Table 4.

	a_1	a_2	a_3
w_1	$10, 10$	$0, 0$	$8, 8$
w_2	$0, 0$	$10, 10$	$8, 8$

Table 4: scalar implicature

There are three information states in this game:

$$\begin{aligned} i_1 &= \{w_1\} \\ i_2 &= \{w_2\} \\ i_3 &= \{w_1, w_2\} \end{aligned}$$

I assume that $p^*(w_1) = p^*(w_2) = \frac{1}{2}$, $\xi(i_1|w_1) = \frac{2}{3}$, $\xi(i_3|w_1) = \frac{1}{3}$, $\xi(i_2|w_2) = \frac{1}{2}$, and $\xi(i_3|w_2) = \frac{1}{2}$. Hence $q^*(i_1) = \frac{1}{3}, q^*(i_2) = \frac{1}{4}$, and $q^*(i_3) = \frac{5}{12}$.

The expected utilities for the three information states are given in Table 5.

	a_1	a_2	a_3
i_1	$10, 10$	$0, 0$	$8, 8$
i_2	$10, 10$	$0, 0$	$8, 8$
i_3	$4, 4$	$6, 6$	$8, 8$

Table 5: utilities relativized to information states

As there are three information states and three messages, strategies can be represented as triples of numbers, similar to the previous example. There is just one honest and rational sender strategy here, namely 123. To calculate the the pragmatically rationalizable strategies, we have to find the smallest H_ω again:

$$\begin{aligned} H_0 &= \{123\} & BR_r(H_0) &= \{123\} \\ H_1 &= \{123, 133\} & BR_r(H_1) &= \{123, 122\} \\ H_2 &= H_1 \end{aligned}$$

So we have

$$J = (\{123, 133\}, \{123, 122\}).$$

J is an SRE. In this equilibrium, Sally always says everything she knows if she is in i_1 or i_3, but she may either be maximally specific in i_2 (using m_2), or she may choose the cheaper but less specific message m_3. Robin will always interpret m_1 and m_2 literally. m_3 may either be interpreted literally as $\{w_1, w_2\}$, or it may be pragmatically strengthened to $\{w_2\}$. If Robin believes that Sally is well-informed (i.e. she is either in i_1 or in i_2), he will interpret m_3 as $\{w_2\}$. If he considers it sufficiently likely that she is not very well-informed, he will not make this implicature.

This example illustrates how the notion of pragmatic rationalizability captures a pragmatic reasoning principle that Levinson (2000) called the *Q-Heuristics:* "What isn't said, isn't." If Sally doesn't say explicitly that all boys came to the party, Robin can assume that it is not the case.

Note that the set of pragmatically rationalizable strategies depends on the underlying probability distributions in a subtle way. If we change ξ so such $\xi(i_3|w_1) = \xi(i_3|w_2)$, the set of pragmatically rationalizable strategies turns out to be

$$(\{123, 133, 121\}, \{123, 122\}).$$

Now it is also pragmatically rationalizable for Sally to use message m_1 in state i_3. This may occur in a situation where Sally believes that Robin erroneously believes her to have complete knowledge of the state of the world, while she has only partial knowledge. In this case, every message that Sally can send will have the effect that Robin extracts more information from it than Sally is justified to provide. All that Sally can do in this situation is to do damage control and to send a message that minimizes the loss of utility due to the possible misinformation while keeping costs low. Depending on the expected probabilities in i_3, m_1 might be such a message. In a more realistic model, Sally might choose the option of sending a modalized statement instead, like "Some boys came to the party, perhaps even all."

Levinson discusses a second pragmatic principle, the *I-Heuristics:* "What is simply described is stereotypically exemplified." These two heuristics jointly essentially capture the reasoning principles that, in classical Gricean pragmatics, are formulated as conversational maxims (cf. Grice 1975). Here are a few examples that illustrate the effects of the I-Heuristics.

(3) a. John's book is good. $\rightsquigarrow$ The book that John is reading or that he has written is good.

b. a secretary $\rightsquigarrow$ a female secretary

c. road $\rightsquigarrow$ hard-surfaced road

The notion of "stereotypically exemplification" is somewhat vague and difficult to translate into the language of game theory. I will assume that propositions with a high prior probability are stereotypical. Also, I take it that "simple description" can be translated into "low signaling costs." So the principle amounts to "Likely propositions are expressed by cheap forms."

Here is a schematic example that illustrates how the I-Heuristics is a consequence of pragmatic rationalizability. Suppose there are two worlds, w_1 and w_2, such that $p^*(w_1) = \frac{2}{3}$ and $p^*(w_2) = \frac{1}{3}$. So we have three information states again, $i_1 = \{w_1\}$, $i_2 = \{w_2\}$, and $i_3 = \{w_1, w_2\}$. Let us say that $\xi(i_1|w_1) = \xi(i_2|w_2) = \xi(i_3|w_1) = \xi(i_3|w_2) = \frac{1}{2}$. Hence $q^*(i_1) = \frac{3}{8}$, $q^*(i_2) = \frac{1}{8}$, and $q^*(i_3) = \frac{1}{2}$.

There are three actions: a_1 is optimal in world w_1 (Sally wants to refer to a hard-surfaced road), a_2 in w_2 (soft-surfaced road) and a_3 (waiting for further information) is optimal under the prior probability distribution. The payoffs are given in Table 6. The payoffs are chosen such that they are inversely monotonically related to the amount of information that Robin is still missing. Picking the correct world is optimal (payoff 24), and picking the wrong world is bad (payoff 0). Maintaining the prior distribution, i.e. action a_3, is better in w_1 (payoff 20) than in w_2 because the surprise for Robin when learning the truth is higher in w_2 than in w_1 if he chooses a_3.

	a_1	a_2	a_3
w_1	$24, 24$	$0, 0$	$20, 20$
w_2	$0, 0$	$24, 24$	$16, 16$

	a_1	a_2	a_3
i_1	$24, 24$	$0, 0$	$20, 20$
i_2	$0, 0$	$24, 24$	$16, 16$
i_3	$18, 18$	$6, 6$	$21, 21$

Table 6: I-implicature

There are three messages: $\|m_1\| = \{w_1\}$, $\|m_2\| = \{w_2\}$, and $\|m_3\| = \{w_1, w_2\}$. m_1 and m_2 correspond to precise and complex expressions (like "hard-surfaced road" and "soft-surfaced road"), while m_3 is a simple but less precise message (like "road"). So we assume again that there are two cost functions, with $c_1(m_1) = c_1(m_2) = 5$, and $c_1(m_3) = c_2(m_k) = 0$ for all k.

Here is the computation of the pragmatically rationalizable equilibrium:

$$\begin{array}{lcllcl} H_0 & = & \{123\} & BR_r(H_0) & = & \{123\} \\ H_1 & = & \{123, 323\} & BR_r(H_1) & = & \{121, 123\} \\ H_2 & = & H_1 & & & \end{array}$$

so J, which is an SRE, turns out to be

$$J = (\{123, 323\}, \{123, 121\}).$$

So it is pragmatically rationalizable for Sally to be perfectly honest, or to use the underspecified m_3 in i_1 (if she deems to costs of using m_1 as too high). Robin will consider m_3 as pragmatically ambiguous, denoting either $\{w_1\}$ or $\{w_1, w_2\}$.

5 Connection to Optimality Theory

In a series of publications, Reinhard Blutner and co-workers have developed a new model of formal pragmatics (see for instance Blutner 2001; Jäger 2002; Blutner et al. 2006). It is based on Prince and Smolensky's (1993) Optimality Theory (OT), but it combines OT's constraint-based approach to compute preference with a novel evaluation procedure, which leads to *Bidirectional Optimality Theory* (BiOT).

The paradigmatic application of BiOT is a kind of pragmatic reasoning that Levinson (2000) calls *M-implicature*. It can be illustrated with the following example:

(4) a. John stopped the car.

b. John made the car stop.

The two sentences are arguably semantically synonymous. Nevertheless they carry different pragmatic meanings if uttered in a neutral context. (4a) is preferably interpreted as *John stopped the car in a regular way, like using the foot brake.* This would be another example for the I-heuristics. (4b), however, is also pragmatically strengthened. It means something like *John stopped the car in an abnormal way, like driving it against a wall, making a sharp u-turn, driving up a steep mountain, etc.*

This pattern is predicted by BiOT. Informally, the reasoning can be described as follows: If John stopped the car in a regular way, (4a) is an efficient and truthful statement for Sally. Also, if Robin assumes (4a) to be true, it is a good guess that John actually stopped the car in a regular way because this is the most likely scenario. So (4a) is to be pragmatically strengthened to "John stopped the car in a regular way." If John stopped the car in an irregular way, (4a) would therefore be misleading. Therefore Sally will consider (4b) instead, because this is the cheapest non-misleading message. Likewise, Robin will infer from hearing (4b) that (4a) was apparently inappropriate (because otherwise Sally hat chosen it). So if (4b) is true and the implicature of (4a) is false, John must have stopped the car in an irregular way. Hence (4b) is pragmatically strengthened to "John stopped the car in an irregular way."

To replicate this kind of reasoning in the present setup, the belief revision policy that is captured in Definition 2 has to be refined. The problem is that the more complex message (4b) is never sent in the honest sender strategy h because there is a less costly synonym. Therefore its probability is 0 for every probability distribution over H_0, so neither clause 2 nor clause 3 of Definition 2 are applicable. So as far as the present system goes, Robin might assume any posterior belief upon observing (4b) in $BR(H_0)$. This is much too unrestricted.

Let us have a closer look at this problem. The scenario is essentially similar to the one in example three, with a likely possible world w_1 (where John stopped the car by using the foot brake), a less likely world w_2 (where John stopped the car by making a u-turn), and a receiver who tries to guess which world is the true one. So let us assume that the utilities are as in that example; the utilities are given in Table 6. Let us furthermore assume that $p^*(w_1) = .75$ and $p^*(w_2) = .25$. Unlike in example three, however, I take it that it is common knowledge that Sally has complete knowledge, so $\xi(i_3|w_1) = \xi(i_3|w_2) = 0$, and hence $q^*(i_1) = .75$, $q^*(i_2) = .25$, and $q^*(i_3) = 0$.

Maintaining the rich language assumption, we still assume that there is a message m_1 with $\|m_1\| = \{w_1\}$ ("John stopped the car by using the foot brake."), a message m_2 with $\|m_2\| = \{w_2\}$ ("John stopped the car by making a u-turn"), and an underspecified message m_3 with $\|m_3\| = \{w_1, w_2\}$. Furthermore we now have an additional message m_4 which is synonymous to m_3, i.e. $\|m_4\| = \{w_1, w_2\}$ as well.

The crucial point of this example is that the two specific messages m_1 and m_2 are more expensive than the underspecified messages m_3 and m_4, and that m_4 is more expensive than m_3.

So let us say that there is just one cost function c, with $c(m_1) = c(m_2) = 4$, $c(m_3) = 1$ and $c(m_4) = 2$.

Using the definitions as they are, we have

$$H_0 = \{123\} \qquad BR_r(H_0) = \{1211, 1212, 1213, 1221, 1222, 1223, 1231, 1232, 1233\}$$

Because m_3 only occurs if Sally is in state i_3, which has zero probability, and m_4 does not occur in the range of any element of H_0 at all, any of Robin's possible actions that is a best response to some posterior belief σ_2 is a possible best response to m_3 and m_4, and this holds for all three actions in the present example.

This is too unrestricted. Even though Robin will have to give up his belief that Sally plays a strategy from H_0 in the current game if he observes m_3 or m_4, he will not revise his beliefs in an arbitrary way. I will semi-formally sketch a cautious belief revision policy here that captures the essence of the reasoning process underlying BiOT.

Suppose Robin strongly believes Sally to play a strategy from H_n, and he observes a message m that has probability zero under all probability distributions over H_n. I take it that Robin still tries to maintain his beliefs that (a) Sally is rational to degree n, (b) the literal meaning of m is $\|m\|$, and (c) the relative costs of messages are as predicted by c. He may give up his assumptions about p^* or ξ^*, or he may scale c by a non-negative coefficient.[10]

Applied to the example at hand, observing m_3 can be explained either by giving up the assumption that $\xi(i_3|w_i) = 0$, or by setting all costs to 0. In either case, i_3 would receive a posterior probability 1, and the best response is a_3. As for m_4, a minimal belief revision would require the assumption that the cost of m_4 is not higher than m_3. After revising his belief accordingly, Robin will assign the posterior probability 1 to i_3 as well. So we now have:

$$\begin{array}{rclcrcl} H_0 & = & \{123\} & \qquad & BR_r(H_0) & = & \{1233\} \\ H_1 & = & \{123\} & & & & \end{array}$$

In computing the best response to H_1, the same considerations as above apply, so 1233 is a best response. However, m_3 can also be explained if the costs are scaled such that sending m_1 a rational decision for Sally in H_1 (of this revised game) in state i_1, but not in state i_2. This would be the case if costs are scaled by a factor $k \in [\frac{4}{3}, \frac{8}{3})$. If Robin maintains his assumptions about the prior probabilities but revises his assumptions about the costs in this way, his posterior belief after observing m_3 assigns probability 1 to i_1. Hence 1213 is a best response as well. If costs are scaled by a $\frac{8}{3}$, m_3 is the only optimal message for

[10]You may detect an optimality-theoretic flavor here: Sally's degree-n rationality, the literal meanings of messages and the relative costs of messages are the strongest constraints. The prior probabilities p^* and ξ^* and the cost coefficient are weaker constraints, that are easier to give up, the belief in H_n is still weaker, and the specific probability distribution over H_n that Robin assumes in his σ_1-belief is given up first if it is inconsistent with the observation.

Sally in H_1 in state i_1, while in i_2 both m_2 and m_3 are optimal. Hence Robin will assign a posterior probability in $[.75, 1]$ to i_1 in this case, and the best response is either a_1 or a_3. Finally, if costs are scaled with a factor $k > \frac{8}{3}$, m_3 is the only optimal message in all information states, the posterior belief equals the prior belief, and the optimal response is a_3. Therefore we get

$$BR(H_1) = \{1233, 1213\}$$

From this we derive straightforwardly

$$H_2 = \{123, 323\}$$

Now Robin's posterior upon observing m_3 will assign probability 1 to i_1. Upon observing m_4, he may revise his beliefs by multiplying all costs with 0, which leads to assigning the posterior probability 1 to i_3. However, if he assigns a probability $p > .25$ to the strategy 1213, he may also explain m_4 by multiplying the costs with a coefficient $k \in [4, 16p]$, because with these parameters, m_4 is rational for Sally in H_2 in state i_2. So another possible posterior belief for Robin upon observing m_4 would be one that assigns probability 1 to i_2. So we get

$$BR(H_2) = \{1212, 1213\}$$

Hence

$$\begin{array}{rclcrcl} H_3 & = & \{123, 323, 343, 324\} & \qquad & BR_r(H_3) & = & \{1212\} \\ H_4 & = & H_3 & & & & \end{array}$$

So the set of pragmatically rationalizable strategies comes out as

$$J = (\{123, 323, 343, 324\}, \{1233, 1213, 1212\})$$

So the messages are used and interpreted either according to their literal meaning or according to the M-implicature. Even stronger, if the player grant each other a level of sophistication of at least 3, the M-implicatures arise with necessity.

On an intuitive level, the interpretation of m_3 as i_1/a_1 comes about because a possible explanation for the unexpected message m_3 is that Sally is lazy and prefers the cheap m_3 over a more precise message if the loss in utility is not too high. This is more likely to be the case in i_1 than in i_2. This taken into account, a possible explanation for m_4 might be that Sally is lazy enough to prefer a cheap and vague message over a costly and precise one, but she cares enough about precision that it's worth paying the differential costs between m_3 and m_4 if this prevents the risk of severe misunderstandings. This can only happen if m_3, in its pragmatically strengthened interpretation, is seriously misleading, as in state i_2.

The connection between BiOT and game theoretic reasoning that this example suggests is perhaps not that surprising after all. BiOT is concerned with the simultaneous evaluation of sender preferences and receiver preferences, and the two perspectives are recursively intertwined. This is a version of strategic reasoning that can be modeled by game-theoretic means (a point which has already been noticed and exploited in Dekker and van Rooy 2000). As the example also illustrates, the specifics of Blutner's bidirectional evaluation procedure tacitly incorporate certain background assumptions about the mutual epistemic modeling of sender and receiver, and on a certain belief revision policy. It seems likely that various variants of optimality theoretic pragmatics can be embedded into the game theoretic model in a way that reduces differences in the evaluation procedure to different epistemic attitudes of the interacting agents. The general connection of these two frameworks is still largely unexplored terrain though.

6 Conclusion

The topic of this paper was the issue how rational communicators will communicate with signals that have a commonly known exogenous meaning. Since this is essentially the question that Gricean pragmatics is concerned with, the game theoretic approach can be seen as an attempt to formalize the Gricean program.

The basic intuition underlying the proposal made in this paper can be summarized as: *A rational sender will be honest and maximally informative unless she has reasons to do otherwise.*[11] "Reasons to do otherwise" are justified beliefs that the honest and informative strategy is sub-optimal. If it is common knowledge that the sender follows this principle, a rational receiver can rely on it, a rational sender can rely on the receiver relying on it etc.

The particular solution concept proposed here is in a sense hybrid, combining a cumulative notion of *iterated best response*—starting from some salient set of strategies—with an equilibrium notion.

The present proposal, as well as the related approaches mentioned above, are partially programmatic and open up a series of questions for further research. I would like to conclude with pointing out three issues that promise to be especially fertile:

- The details of of the belief revision policy of the receiver (and the sender's assumptions about it) turned out to play a massive role in determining pragmatic inferences. This connection has only been tapped on so far and deserves further scrutiny. Belief revision plays a central role in many pragmatic phenomena beyond the scope of (neo-)Gricean pragmatics. To mention just a few: (a) Presupposition accommodation works because a naive listener assigns probability zero to a presupposing expression if its presupposition is not part of the common ground. This will lead to belief revision, and this can be exploited by a strategic speaker. (b) Rhetorical questions are utterances that are inappropriate in the actual utterance context. A naive listener will adapt his assumptions about the context in a way that makes the question appropriate. This can be anticipated by a strategic speaker and exploited for a rhetorical effect. (c) Non-literal interpretations like active metaphors or indirect speech acts arise because a signal would be irrational in the actual utterance situation, which in turn triggers a belief revision on the side of the hearer.

[11] It is perhaps noteworthy that similar ideas are being pursued in philosophical circles in the context of the epistemology of testimony; cf. for instance Burge (1993). I thank Sanford Goldberg and Matthew Mullins for pointing this connection out to me.

- When children acquire a language, they only observe the usage of signals in actual situations, i.e. their pragmatic usage, not their underlying literal meaning. So in language learning, the semantics has to be inferred from the pragmatics. Since the mapping from the conventionalized semantics to a pragmatically rationalizable equilibrium is usually many-to-one, this problem is not perfectly solvable. If the learner only observes a small sample of game situations, mistakes in meaning induction become even more likely. Under iterated learning, this leads to a dynamics of language change (cf. Kirby and Hurford 2001). A possible direction of change is the semanticization of pragmatic inferences. A formal and computational investigation of the iterated learning dynamics of the game theoretic model, and its comparison with the findings from historical semantics, seems to be a promising undertaking.

- The computation of pragmatically rationalizable equilibria proceeds by the iterated computation of the set of best responses, starting from the set of honest strategies. It seems initially plausible that actual humans use a similar strategy, but only compute a bounded, perhaps small, number of iterations. This leads to the prediction that the cognitive effort of a pragmatic inference is correlated with the number of iterations according to the iterated best response algorithm. This hypothesis is empirically testable via psycholinguistic experiments.

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