Anisotropic Ocular Drift

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With lots of help from Janis Intoy

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Overview

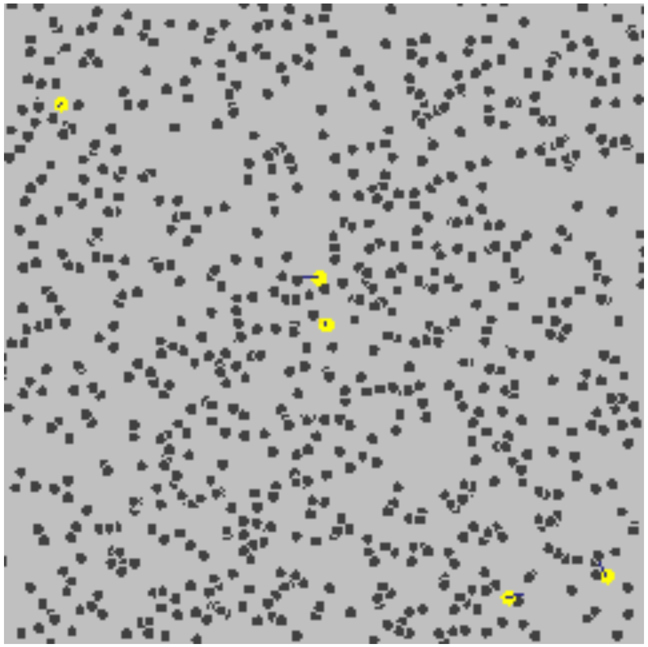
1. Brownian Motion as a model of Ocular Drift
2. Evidence of Optimal Drifts in Certain Tasks
3. Isotropic vs. Anisotropic Optimal Drifts
4. Results from tilted-grating detection and discrimination, Letter task
5. Next steps for investigating anisotropic drift

(Items 3–4: A method for quantifying and comparing the shape of drift)

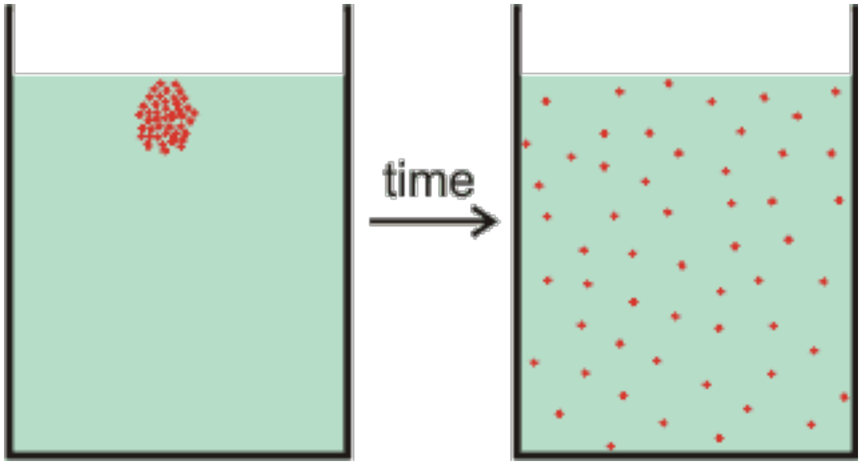
Brownian Motion

- Brownian motion is the apparently random motion of small particles suspended in fluid.
- Brownian motion drives diffusion such that over time particles will move from regions of high concentration to low concentration.
- When diffusion is the same in every direction, i.e. *isotropic,* movement is characterized by a single diffusion coefficient (D).

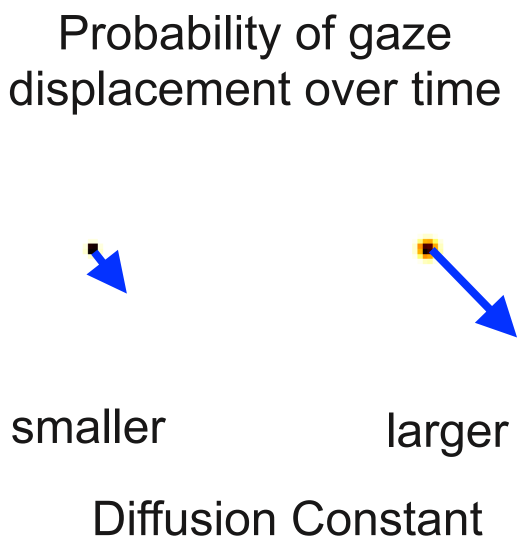
Brownian Motion



Diffusion



Brownian Motion: Applied to Drift

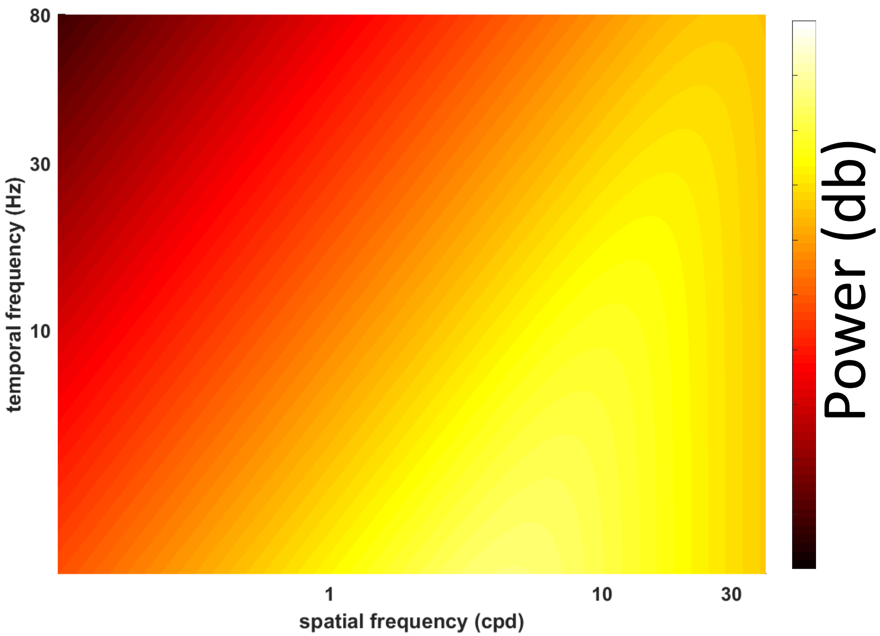


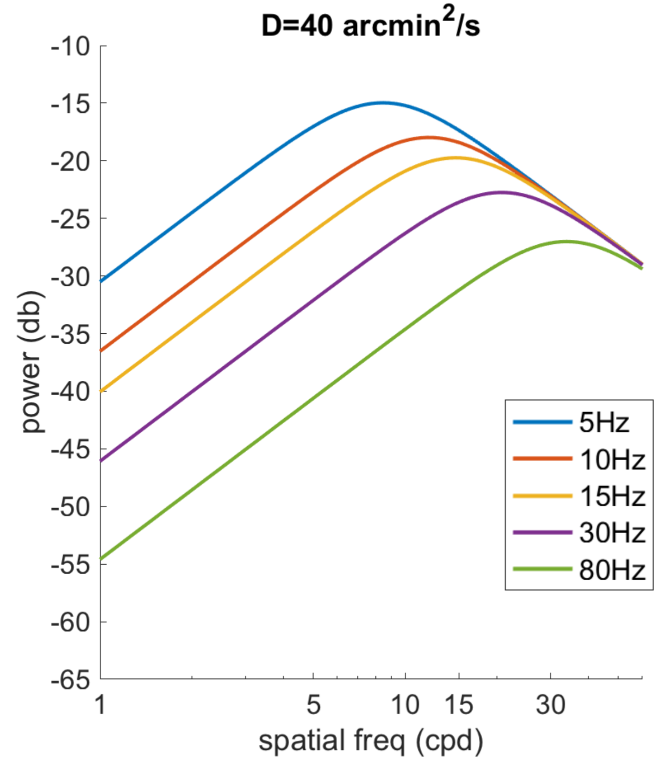
- Parameterized by the diffusion constant, *D*
- Gaze displacement at any given time follows the normal distribution
  - Variance increases **linearly** with time
  - $\langle r^2 \rangle = 4Dt$

Brownian Motion: Applied to Drift

Power provided by drift

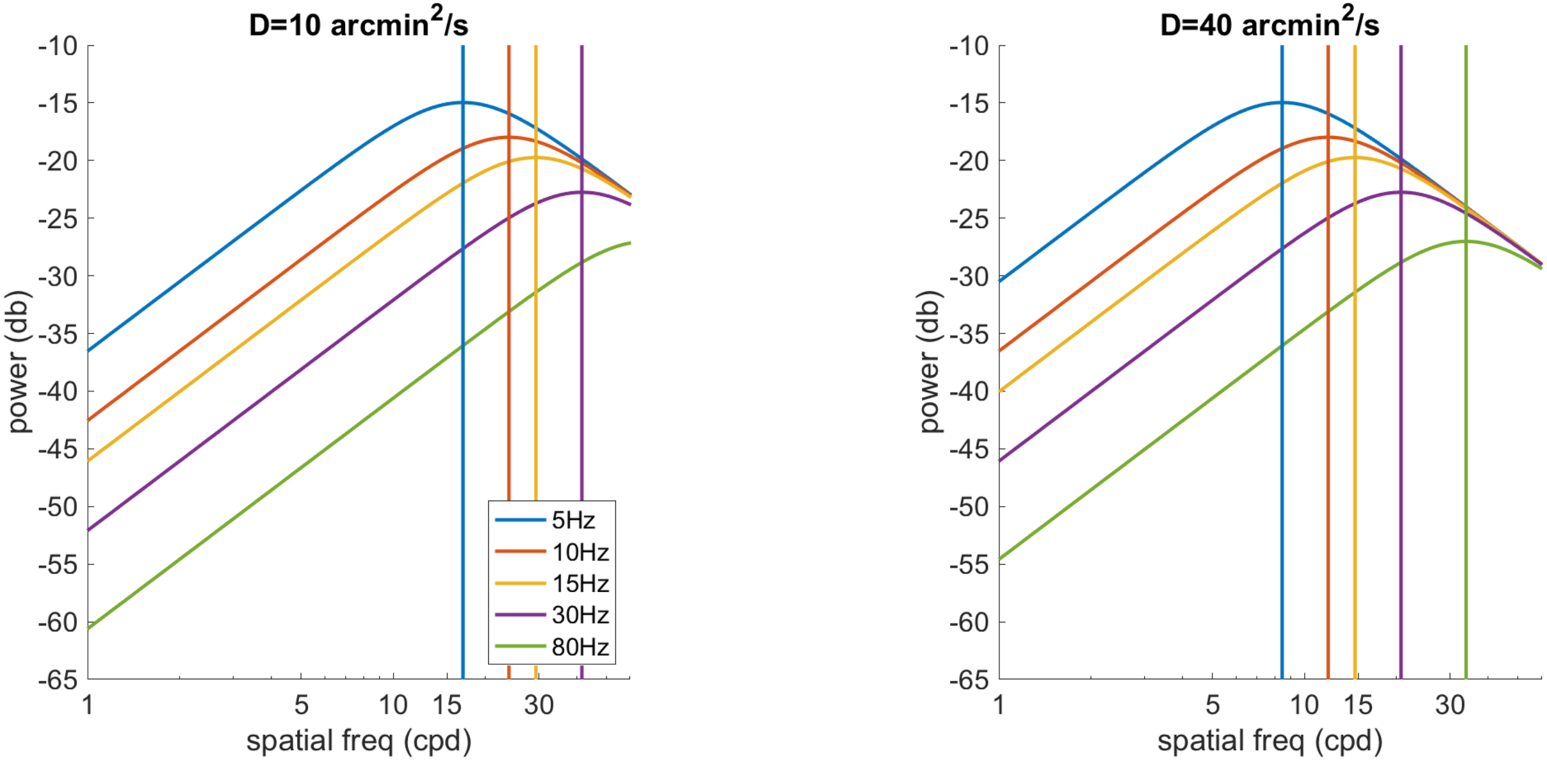
$Q(k, f; D = 40)$





Brownian Motion: Applied to Drift

Power provided by drift



Smaller D --> Increases Critical Frequency

“Optimal” Drifts

Smaller D --> Increases Critical Frequency

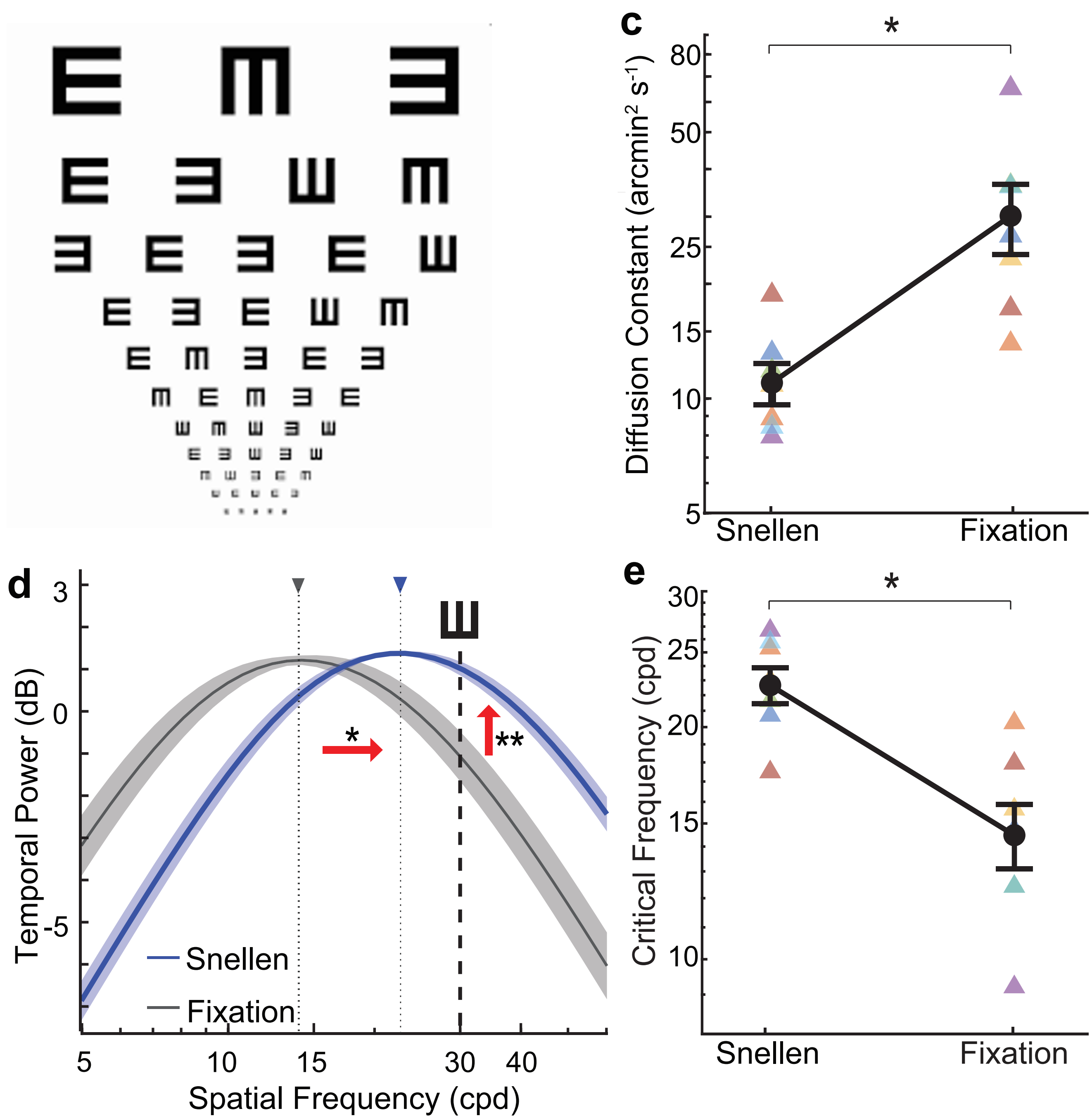
Probability of gaze displacement over time

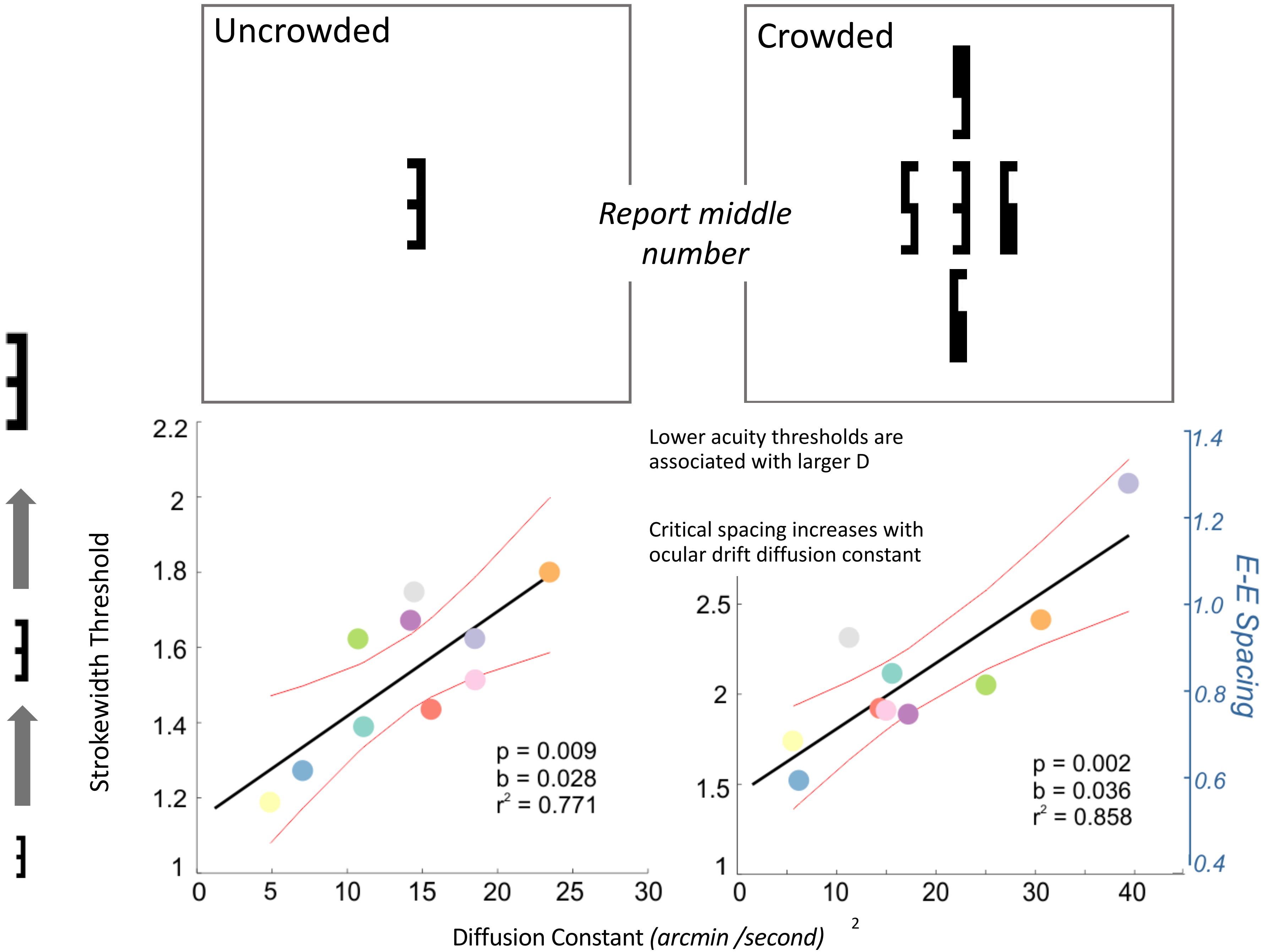
smaller larger

Diffusion Constant

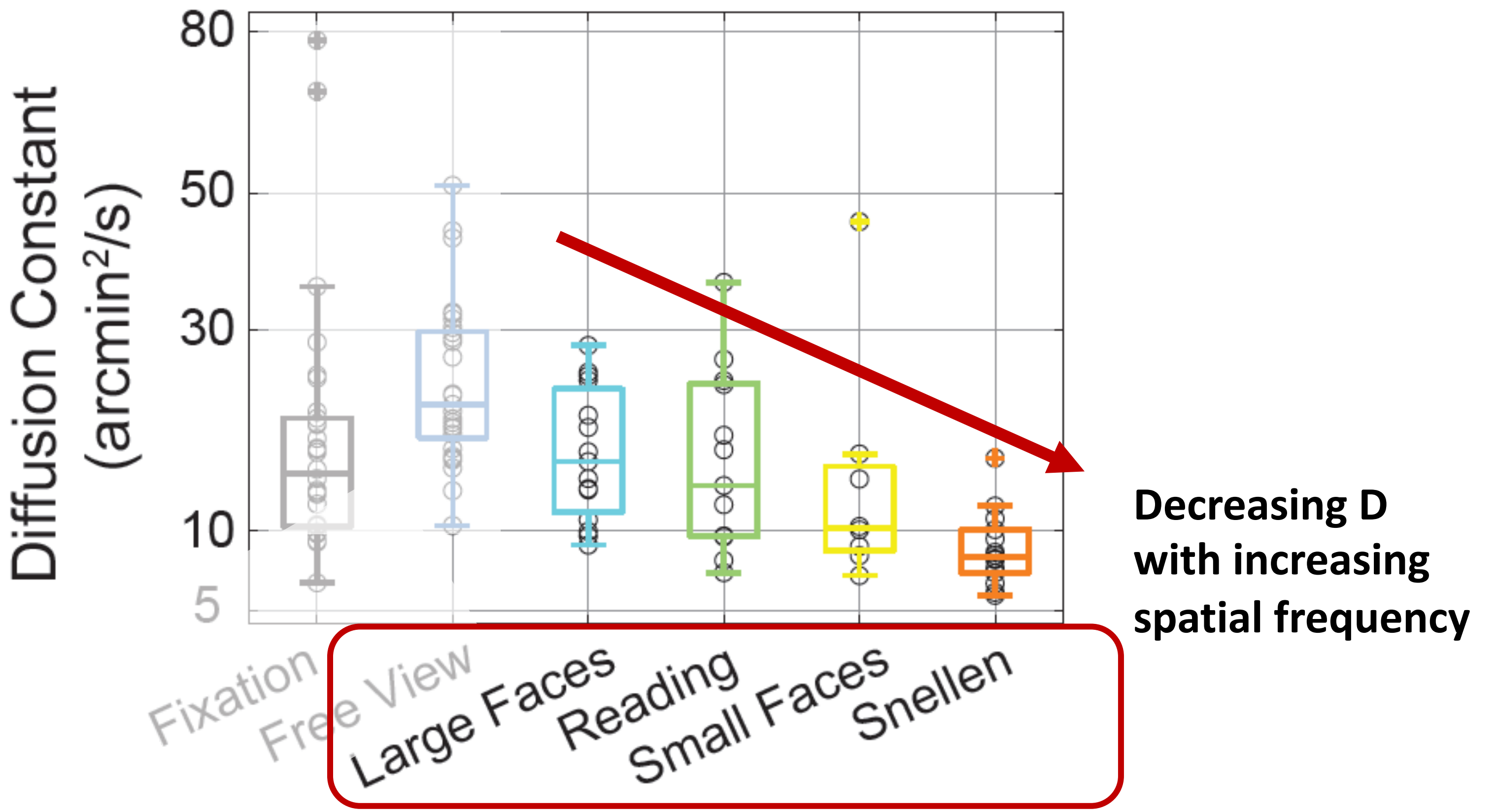
Three Pieces of Evidence:

1. Snellen (Intoy & Rucci, 2020).
2. Crowding (Clark et al, VSS).
3. Drift Meta Analysis (Intoy et al., VSS)


c
*
Diffusion Constant (arcmin² s⁻¹)
80
50
25
15
10
5
Snellen
Fixation
d
3
0
-5
Temporal Power (dB)
*
**
Snellen
Fixation
5
10
15
30
40
Spatial Frequency (cpd)
e
*
Critical Frequency (cpd)
30
25
20
15
10
Snellen
Fixation

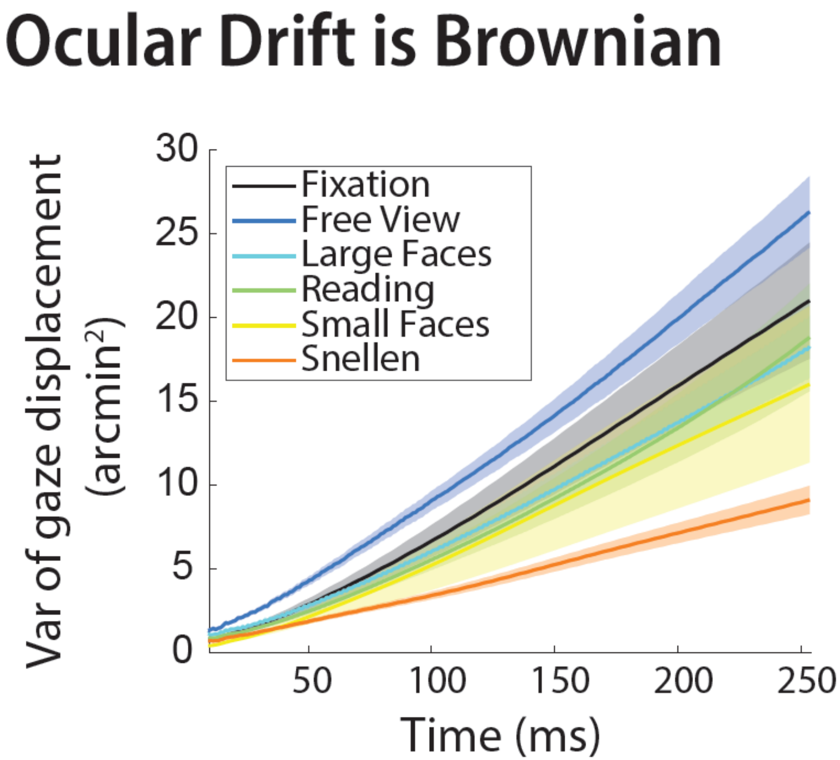

Uncrowded
Crowded
Report middle number
Lower acuity thresholds are associated with larger D
Critical spacing increases with ocular drift diffusion constant
Strokewidth Threshold
E-E Spacing
p = 0.009
b = 0.028
r² = 0.771
p = 0.002
b = 0.036
r² = 0.858
Diffusion Constant (arcmin²/second)

Task-dependent Changes in Drift



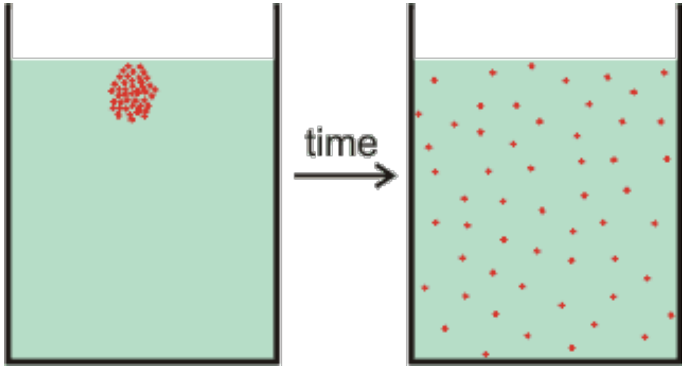
The diffusion constant (which captures changes in speed and curvature) is one parameter by which drift may be controlled.

Is Brownian motion a good model of drift?

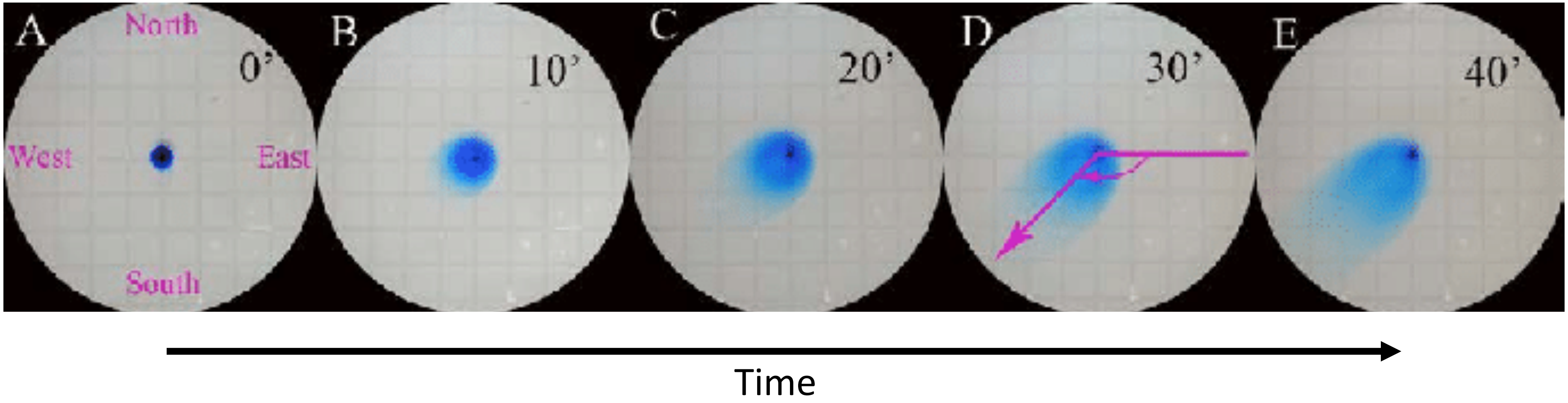


In all tasks, the overall characteristics of ocular drift are compatible with a Brownian motion model. The variance of gaze displacement increases linearly with time, a signature of Brownian motion ($R^2 > 0.95$). However, the diffusion constant, the slope of the increase, varies with task.

Anisotropic Diffusion



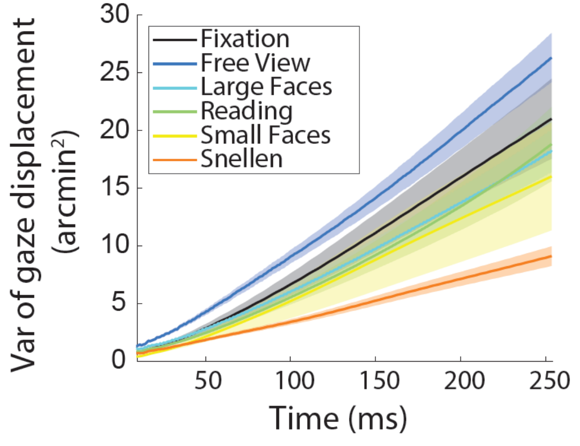
Long-term anisotropic (asymmetric) diffusion of toluidine blue solution in water. Angles indicate the maximum diffusion trend (MDT).



Dai, J. (2014). Macroscopic anisotropic Brownian motion is related to the directional movement of a "Universe field". *Natural Science*, *2014*.

Is Brownian motion a good model of drift?

- Investigate long-term correlations present in drifts.
- BM is uncorrelated in time.
- Fractional Brownian motion (fBM) has a parameter for temporal correlations (Hurst index, H)



Brownian Motion(BM):

$\langle r^2 \rangle = 4Dt$

Fractional Brownian Motion (fBM):

$\langle r^2 \rangle = 4Dt^H$

$H = 1 \rightarrow$ Brownian Motion

Fractional Brownian Motion

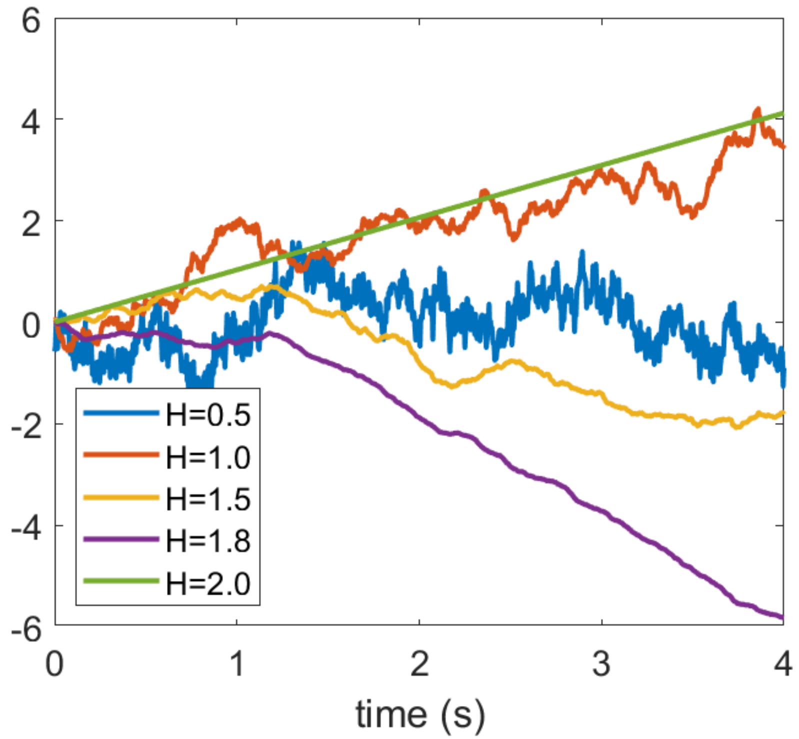
Hurst Index (H)

H > 1 : correlated in time

H = 1: uncorrelated in time (BM)

H < 1: anticorrelated in time

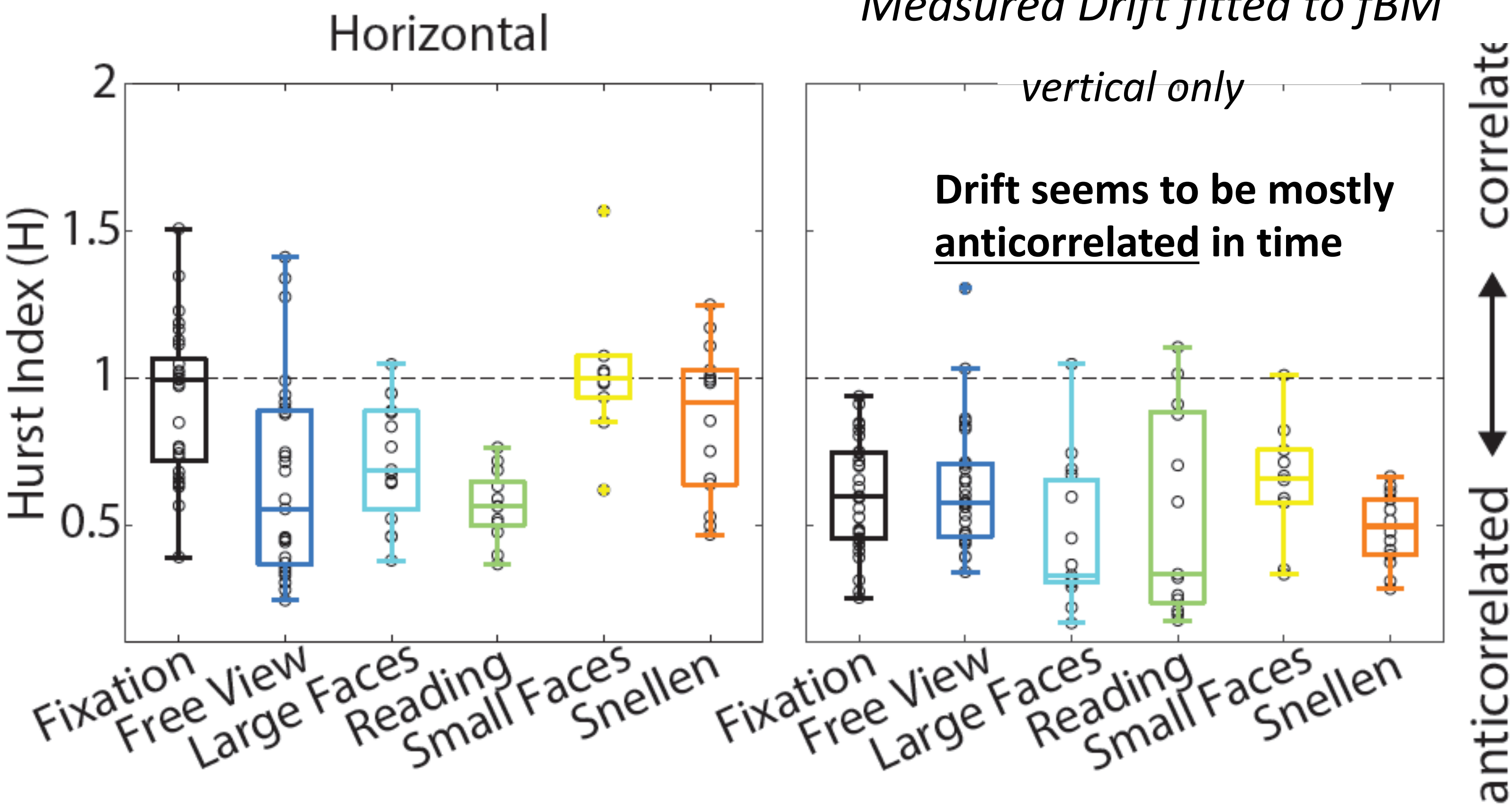
Simulated fBM Drifts



Fractional Brownian Motion

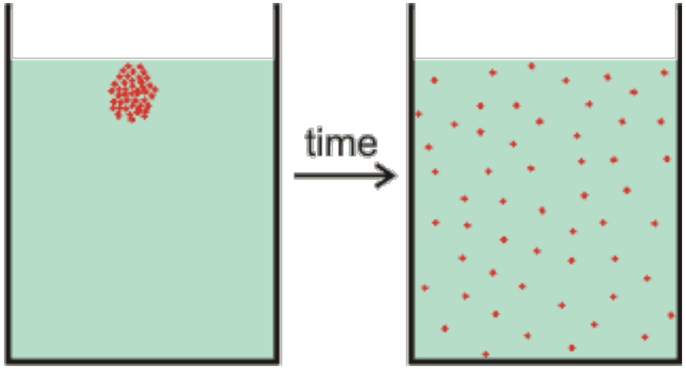
Hurst Index (H)
H > 1 : correlated in time
H = 1: uncorrelated in time (BM)
H < 1: anticorrelated in time

Measured Drift fitted to fBM



Drift seems to be mostly anticorrelated in time

Anisotropic Diffusion



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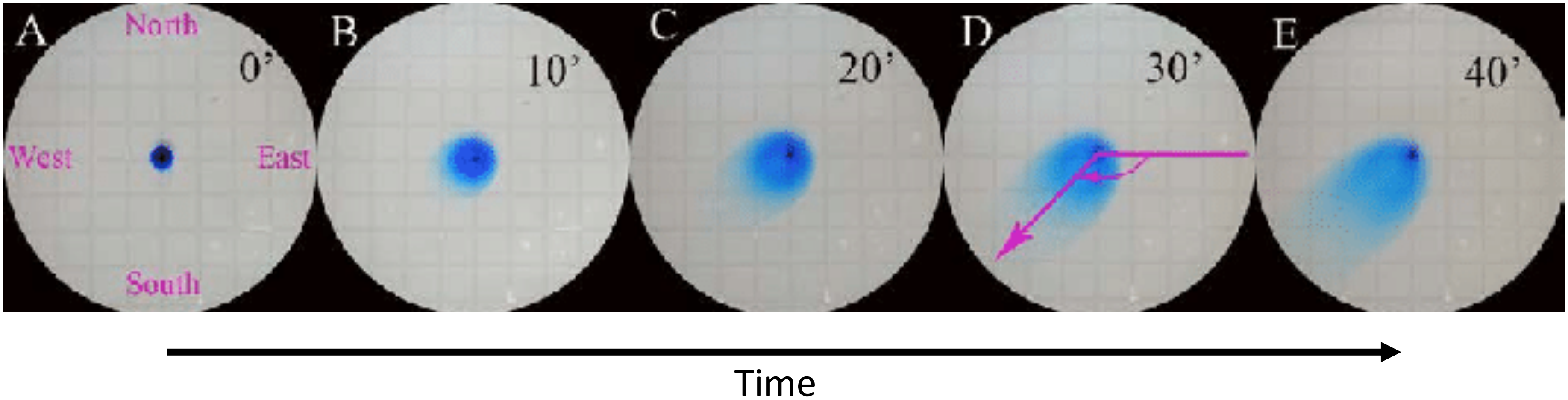


Dai, J. (2014). Macroscopic anisotropic Brownian motion is related to the directional movement of a “Universe field”. *Natural Science*, *2014*.

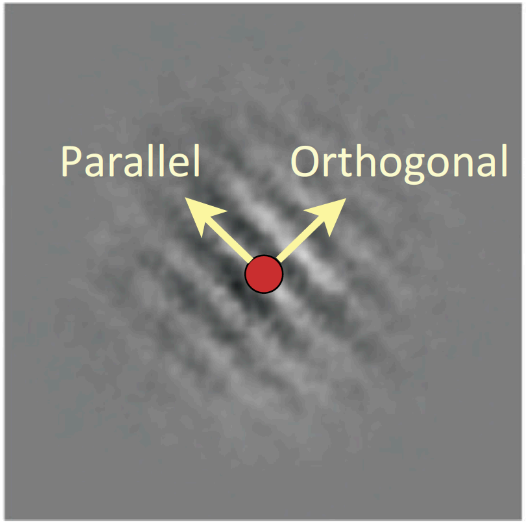
Past and Current Empirical Investigations

Switch Speaker

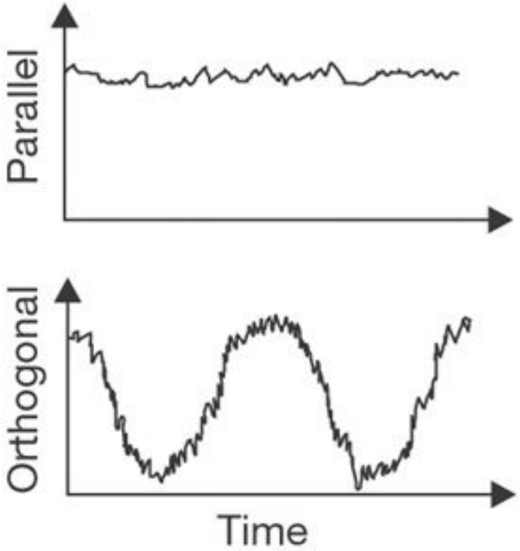
Tasks to Test Stimulus-Dependent Anisotropic Drift

1. Grating detection task
2. Grating discrimination task
3. Letter discrimination task

Optimal Anisotropic Drift?

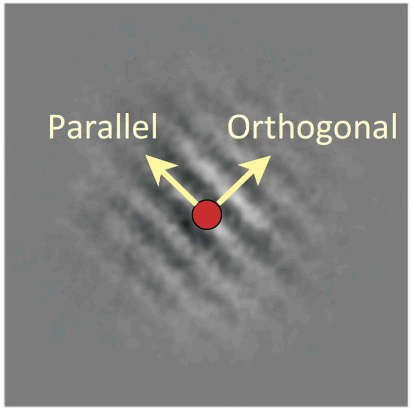


Luminance Transients

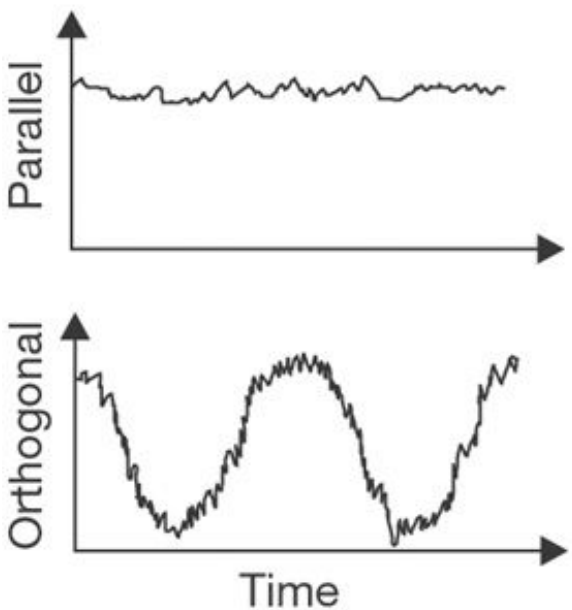


Rucci et al., 2007, Nature

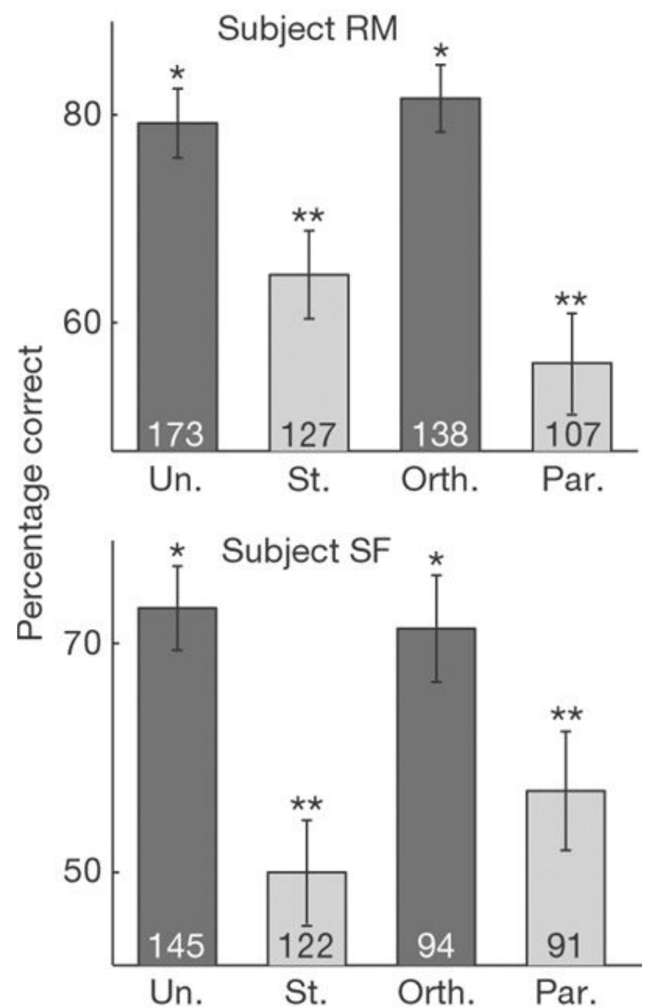
Optimal Anisotropic Drift?



Luminance Transients



Performance Under Partial Stabilization



Rucci et al., 2007, Nature

Expected Results

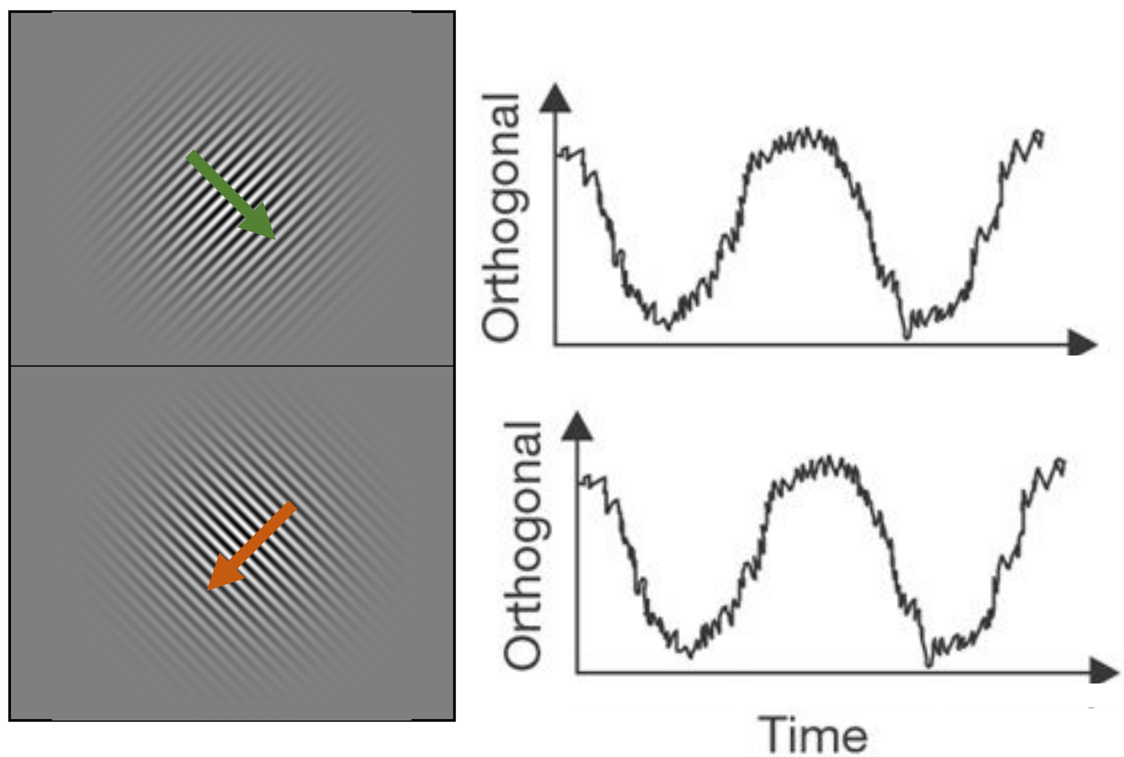
Optimal bias ocular drifts strategy
– from the luminance change point of view

Active, controlled anisotropic FEM

FEM would change for each stimulus

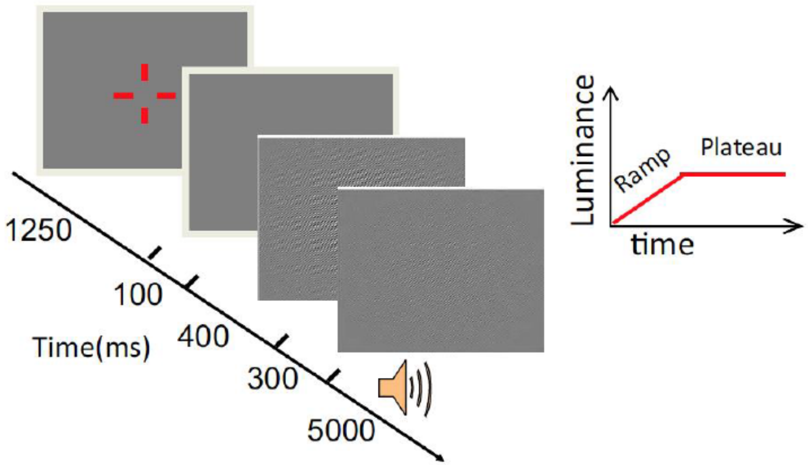
STIMULUS-DRIVEN

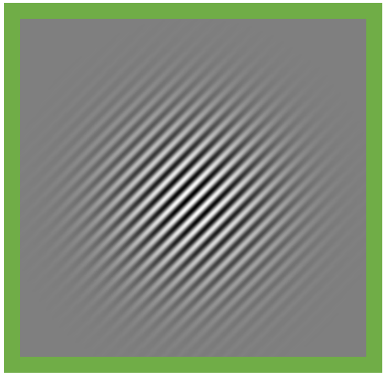
increase luminance
power for all stimuli

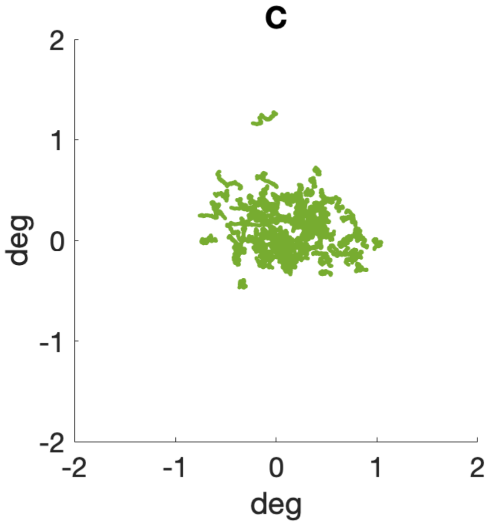


Stimulus-Dependent <u>Anisotropic</u> Drift?

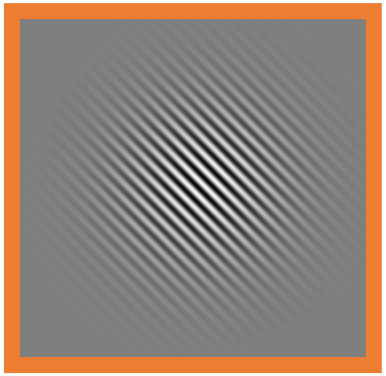
Tilted grating <u>detection</u> task

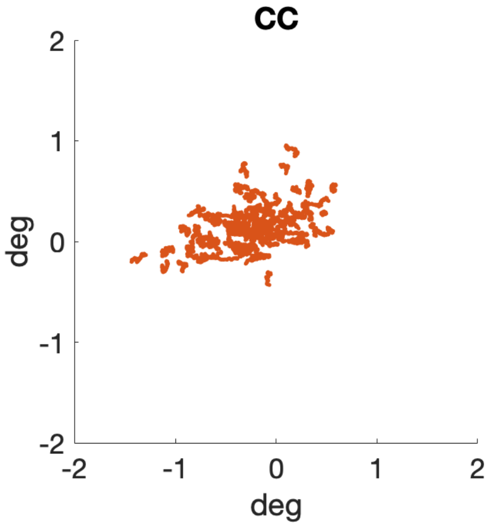






- 8 cpd, 11 deg in size
- Trials are block design; subjects knows the grating orientation
- **Do we see any evidence that the <u>DIRECTION</u> of drift changes with task?**





Tilted Grating Detection:
Previous and Repeated Analyses

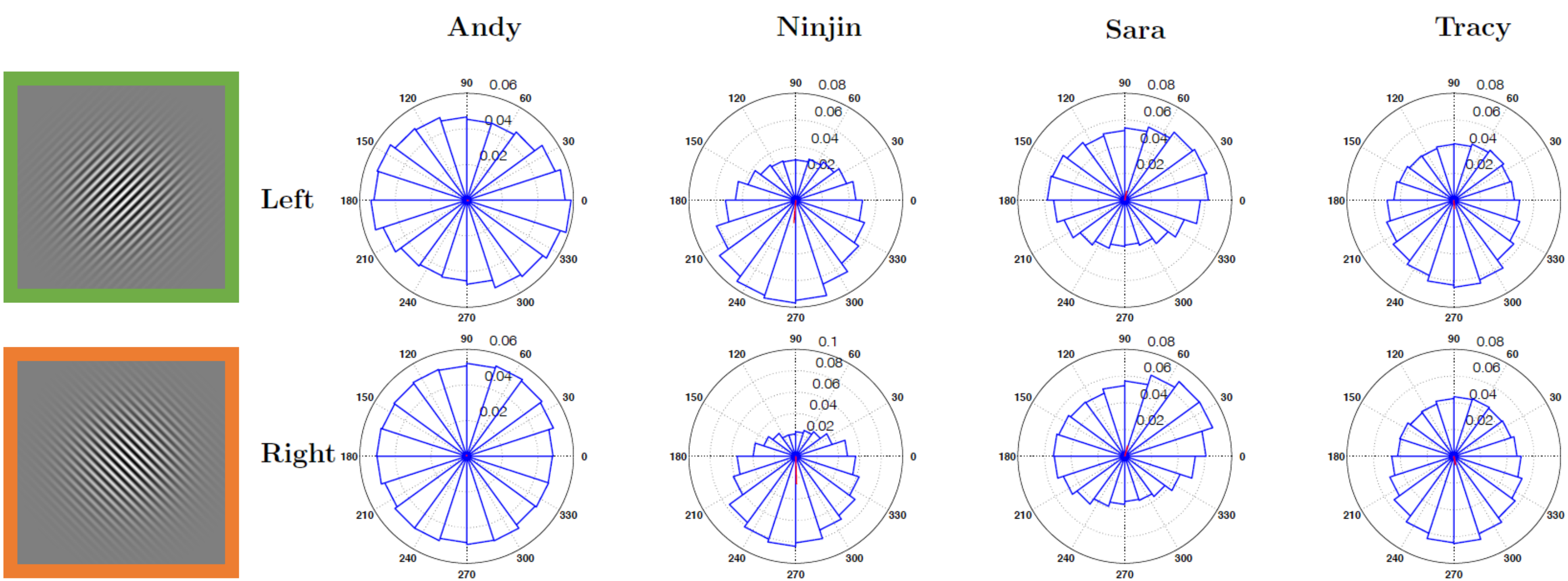
- Drift Metrics
  - Position
  - Velocity
  - Curvature
  - Speed
- Data Segmentation
  - Plateau vs. Drift
  - Fast vs. Slow Drifts
  - Performance
    - Overall detection (d-prime)
    - Hits, Miss, Correct Rejection, False Alarms

In all cases, no consistent differences in metrics between right-tilted and left-tilted sessions

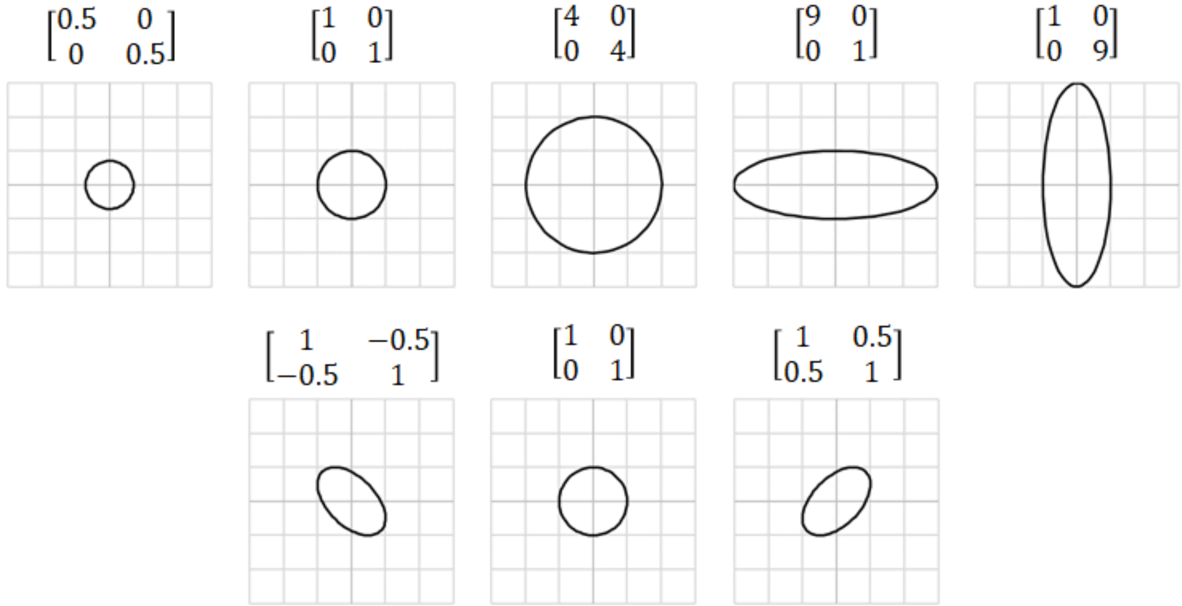
No difference in Velocity Angles between Two Grating Viewing

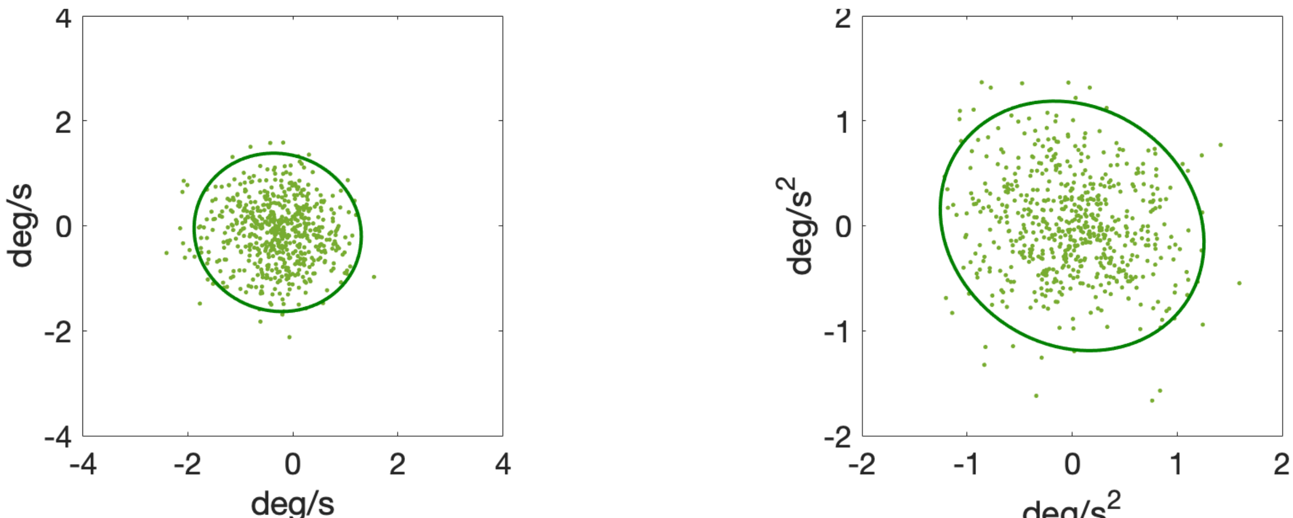
Tilted grating detection task

Histogram of Velocity Angles

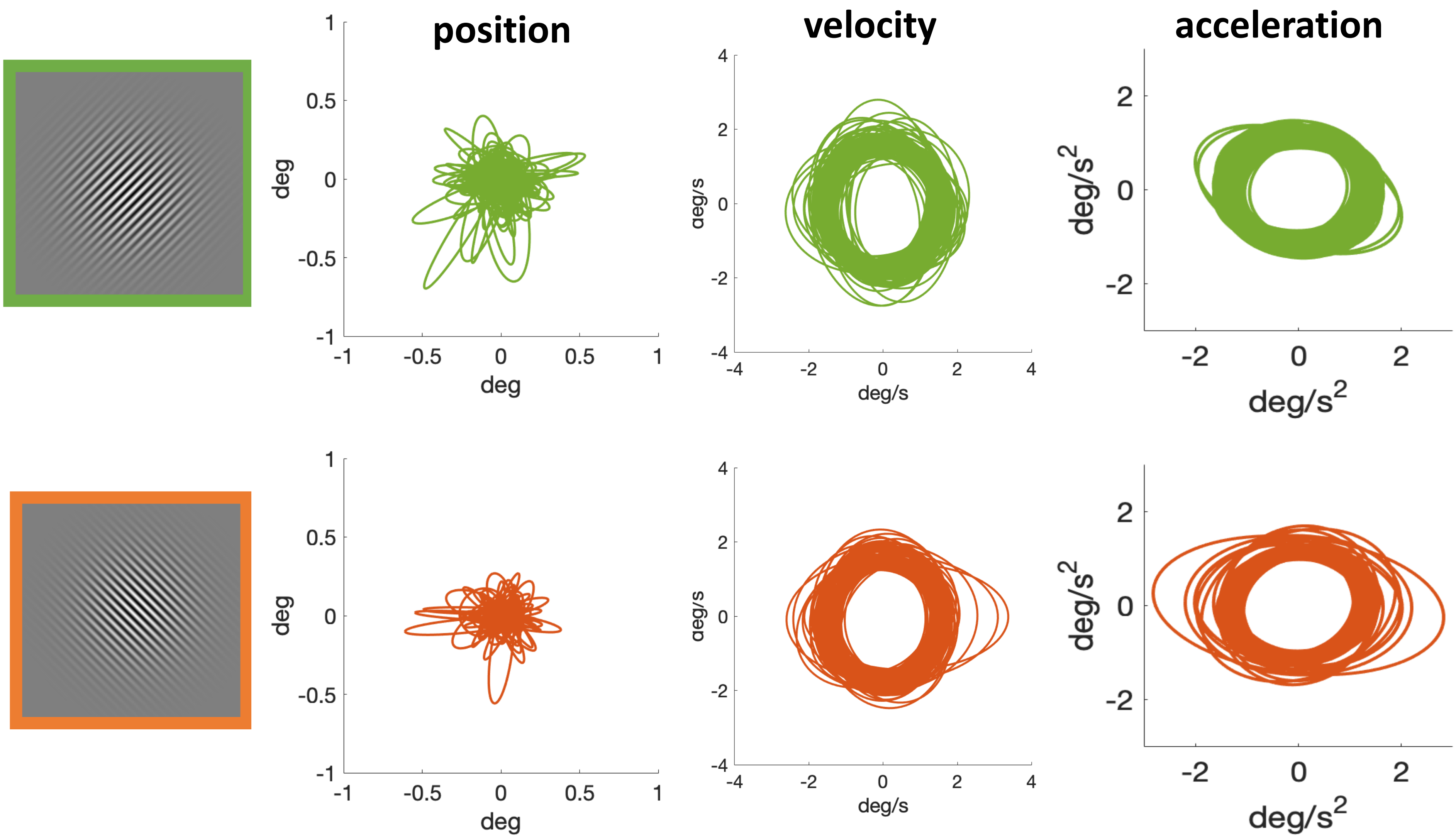


Capturing the 2D Features of Drifts

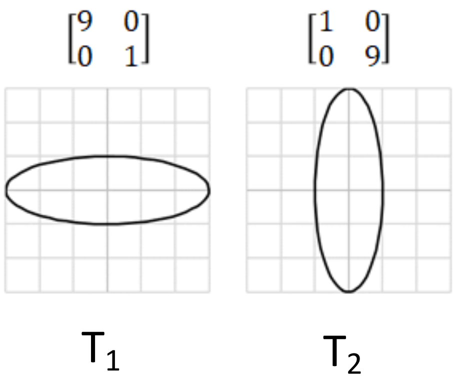


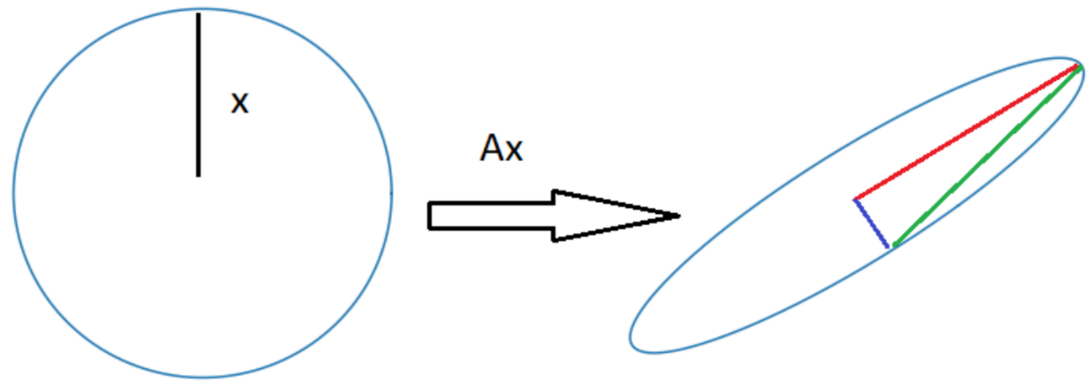


Capture the Features of Drifts on A Trial by Trial Basis



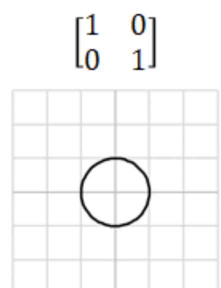
Distance between Two Covariance Matrices





$Q = T_1^{-1} T_2$

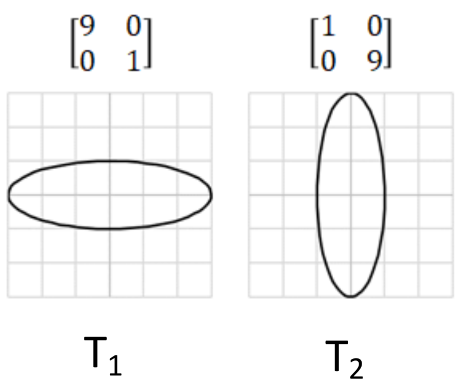
We want to know how close Q is to identity



Eigenvalues: s1
s2

Frobenius norm: sqrt(s1^2 + s2^2)

Distance between Two Covariance Matrices



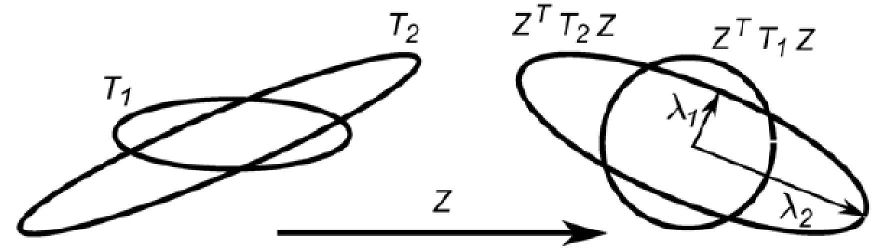


Figure 1. Geometric interpretation of the distance between two tensors. The two structure tensors $\mathcal{T}_1$ and $\mathcal{T}_2$ are represented by the elliptic blobs shown in the lefthand side of the figure. After the change of coordinates defined by the matrix Z, $\mathcal{T}_1$ is represented by the unit disk and the principal axes of $\mathcal{T}_2$ are equal to the eigenvalues λ_1 and λ_2 that appear in (6), see text.
doi:10.1371/journal.pcbi.1000625.g001

$Q = T_1^{-1} T_2$

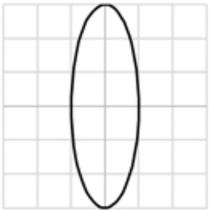
We want to know how close Q is to identity

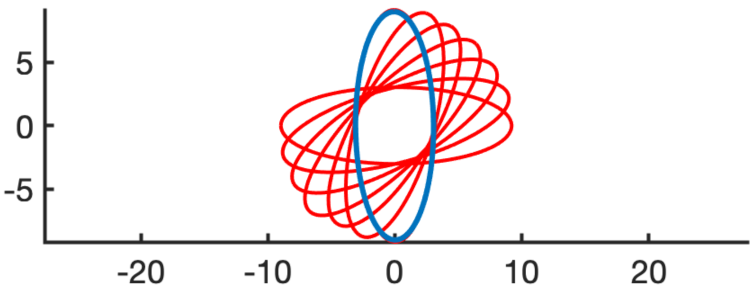
the Frobenius norm of the matrix-log of Q

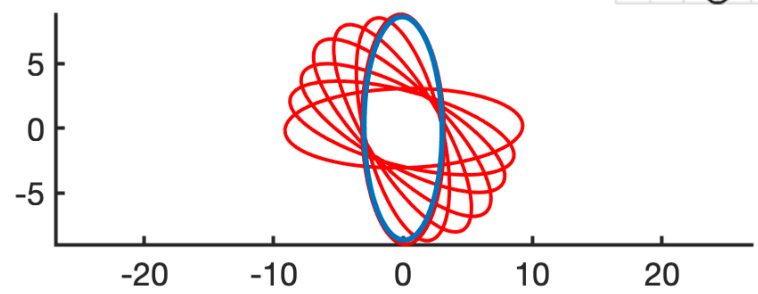
$$d_0(\mathrm{T}_1, \mathrm{T}_2) = \left\| \log \mathrm{T}_1^{-1} \mathrm{T}_2 \right\|_F = \sqrt{\log^2(\lambda_1) + \log^2(\lambda_2)}$$

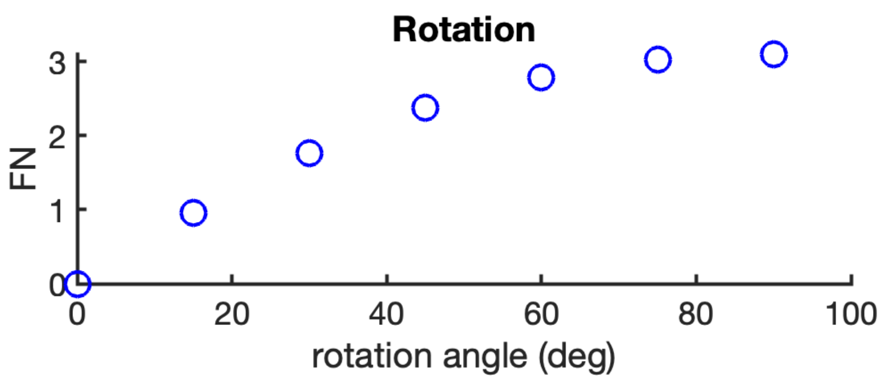
Rotation Changes the Standard Distance between T_1 and T_2

$T_1 \quad \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

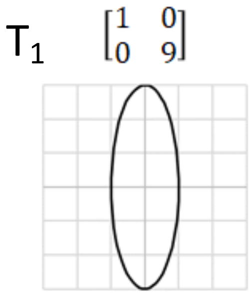




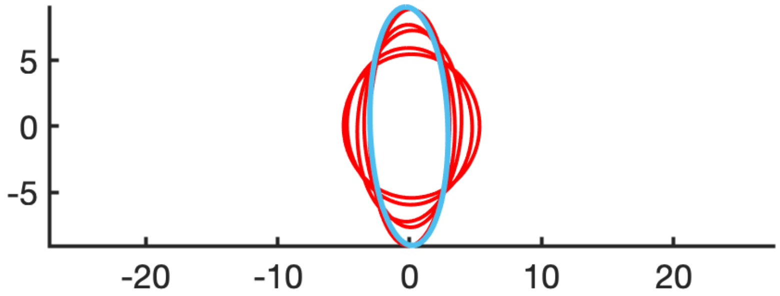




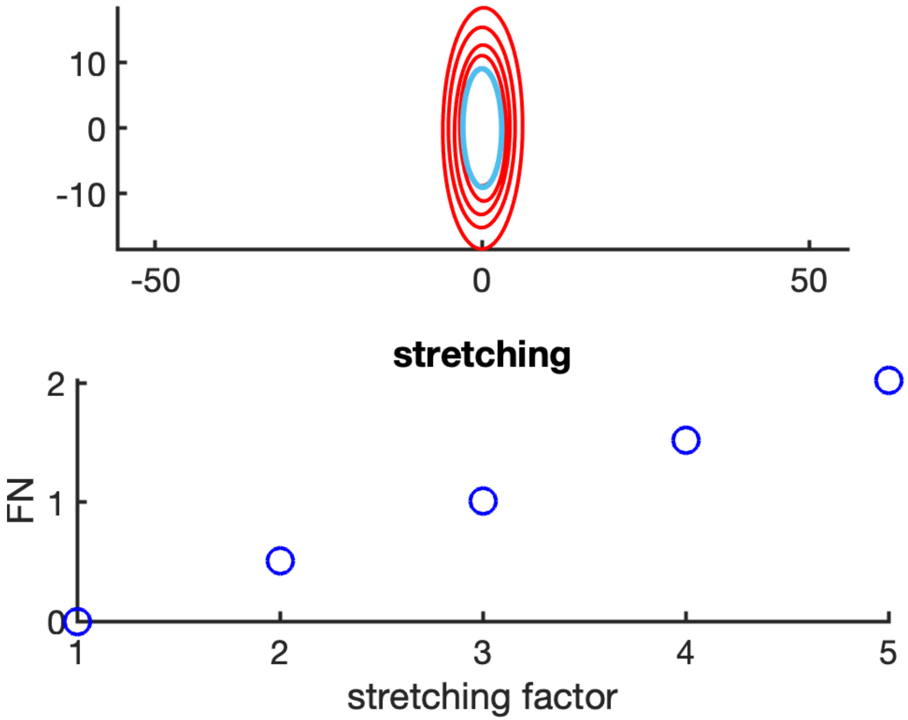
Stretching Change the Standard Distance between T_1 and T_2



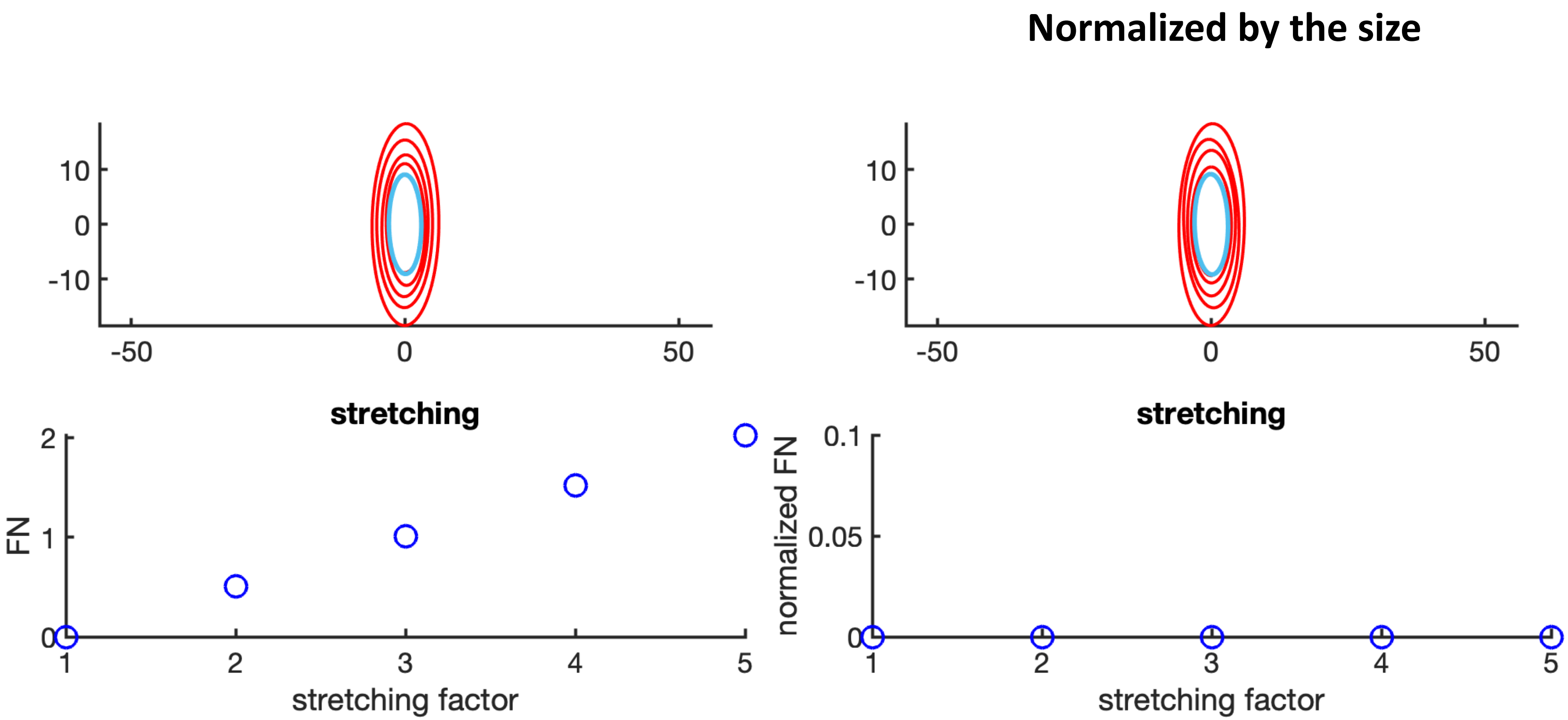
Without changing the size



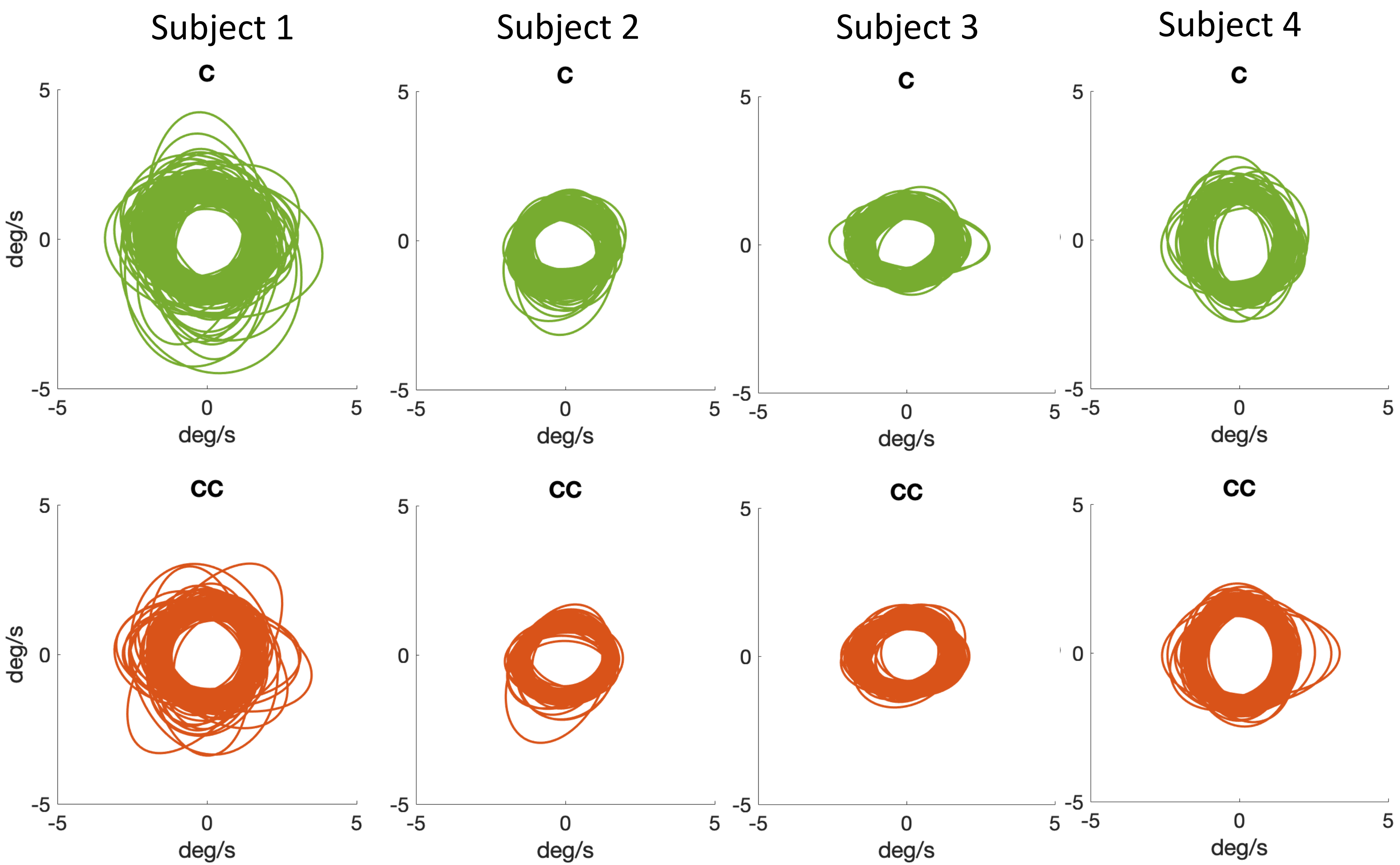
With changing the size



Change in Size Can Be Normalized by A Single Factor



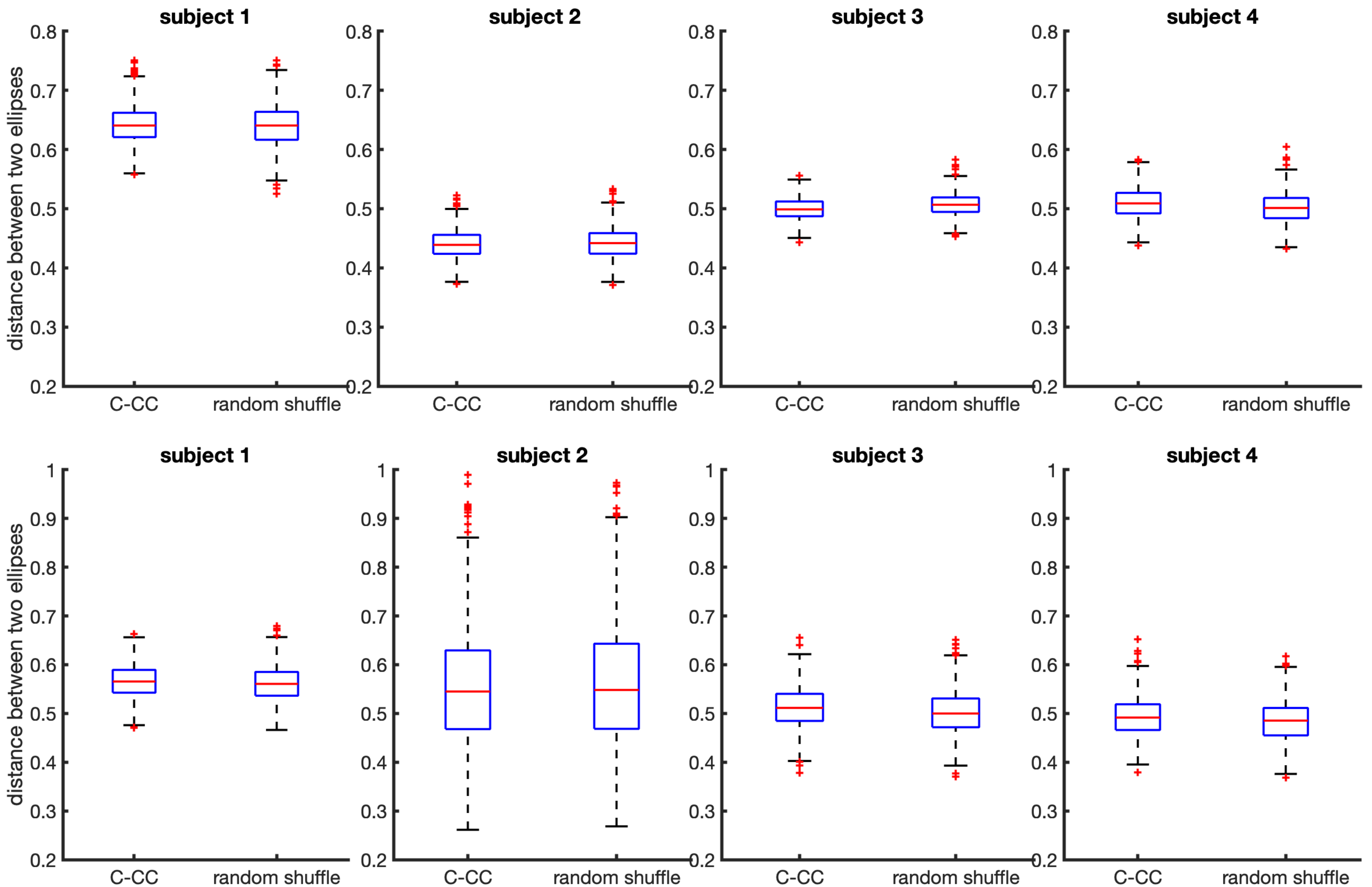
Difference in Directional Bias between Two Grating Viewing?



No Difference in Directional Bias between Two Grating Viewing

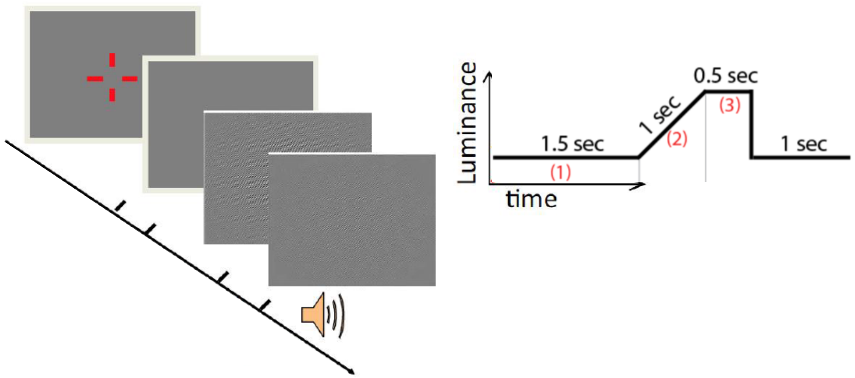
Hit

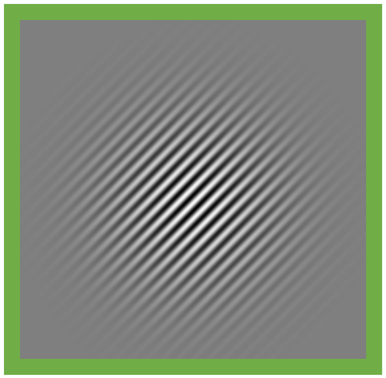
Miss

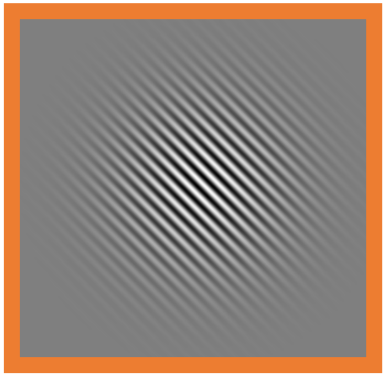


Tasks to Test Stimulus-Dependent <u>Anisotropic</u> Drift

Tilted grating <u>discrimination</u> task

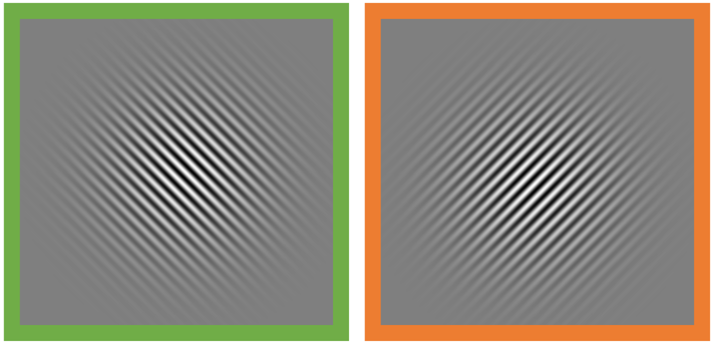






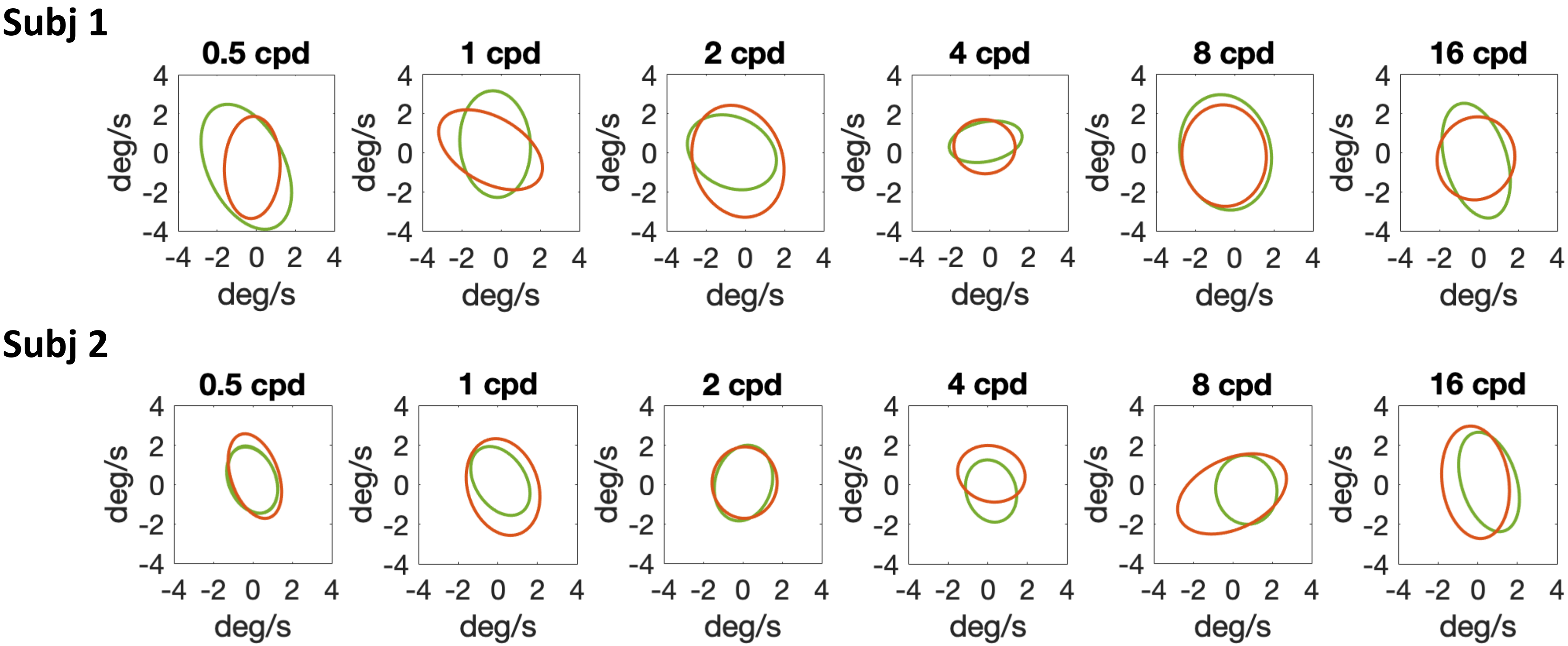
- 0.5 - 16 cpd, 4 deg in size
- Trials are block design; subjects know the spatial frequency of the grating
- **Do we see any evidence that the <u>DIRECTION</u> of drift changes with task?**

Overall Differences between Two Tilted Grating Viewing

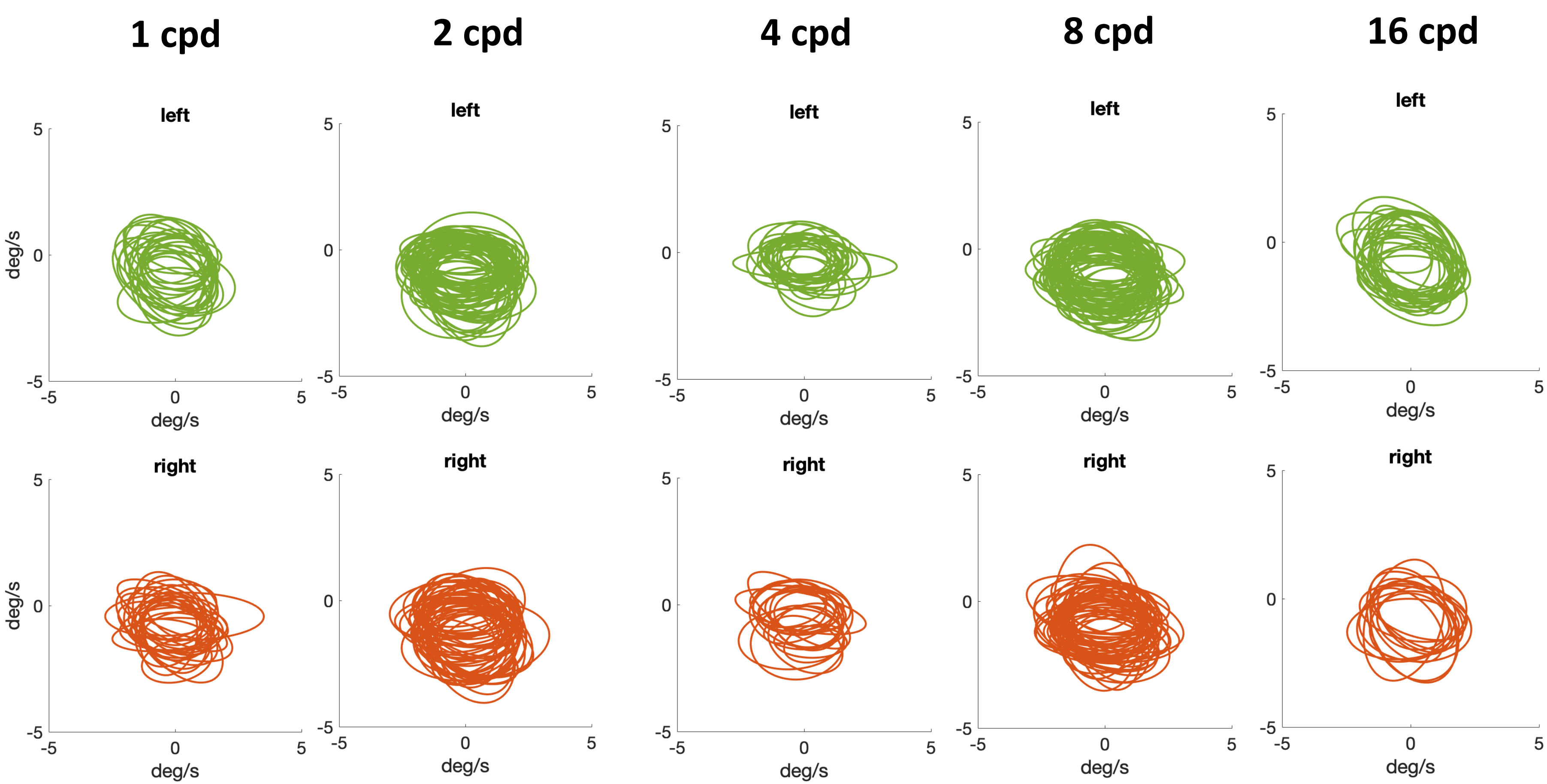


ALL DATA FITTED WITH 1 ELLIPSE

not trial-by-trial



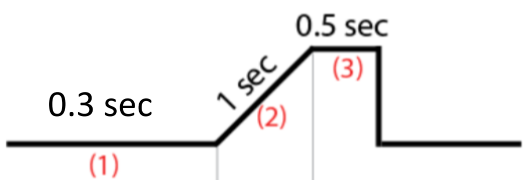
No Trial-by-Trial Differences between 2 Tilted Grating Viewing

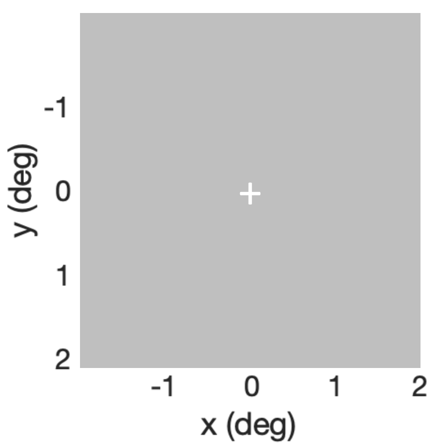


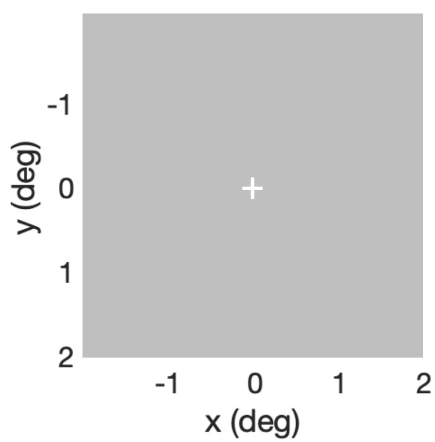
Tasks to Test Stimulus-Dependent Anisotropic Drift

Letter discrimination task

- Letter pairs, ~1.5 deg in size
- Trials are block design; subjects know which letter pairs within trials
- **Do we see any evidence that the DIRECTION of drift changes with task?**

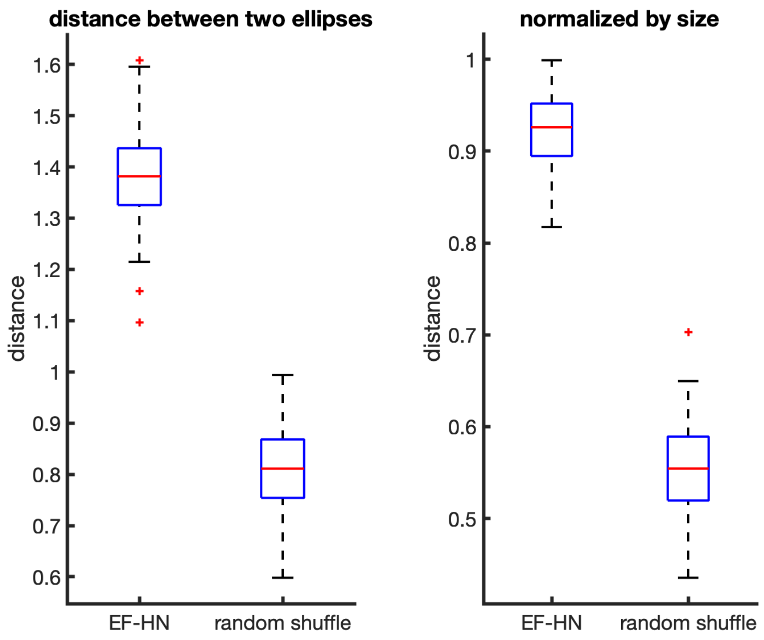


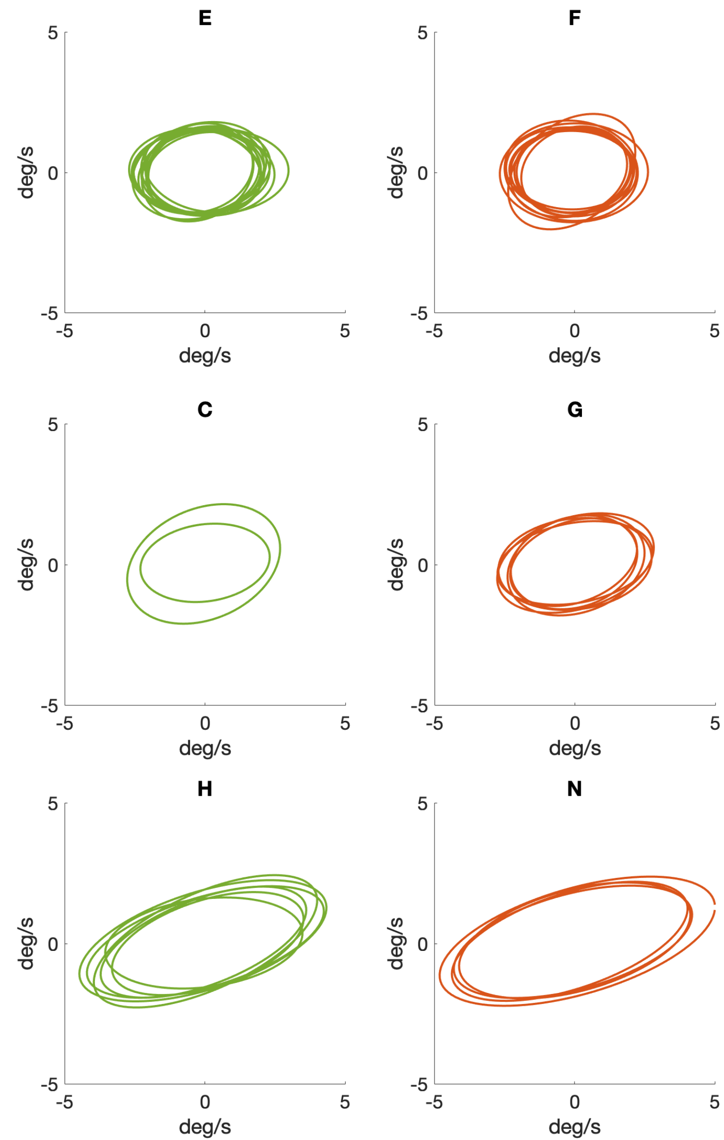




Difference in velocities between pairs

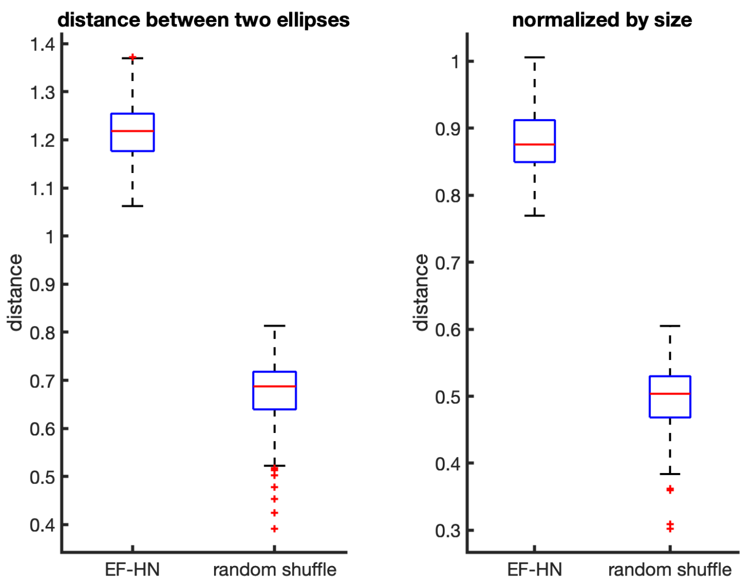
Preliminary data suggests no directional bias between different letters within blocks, but difference between different pairs

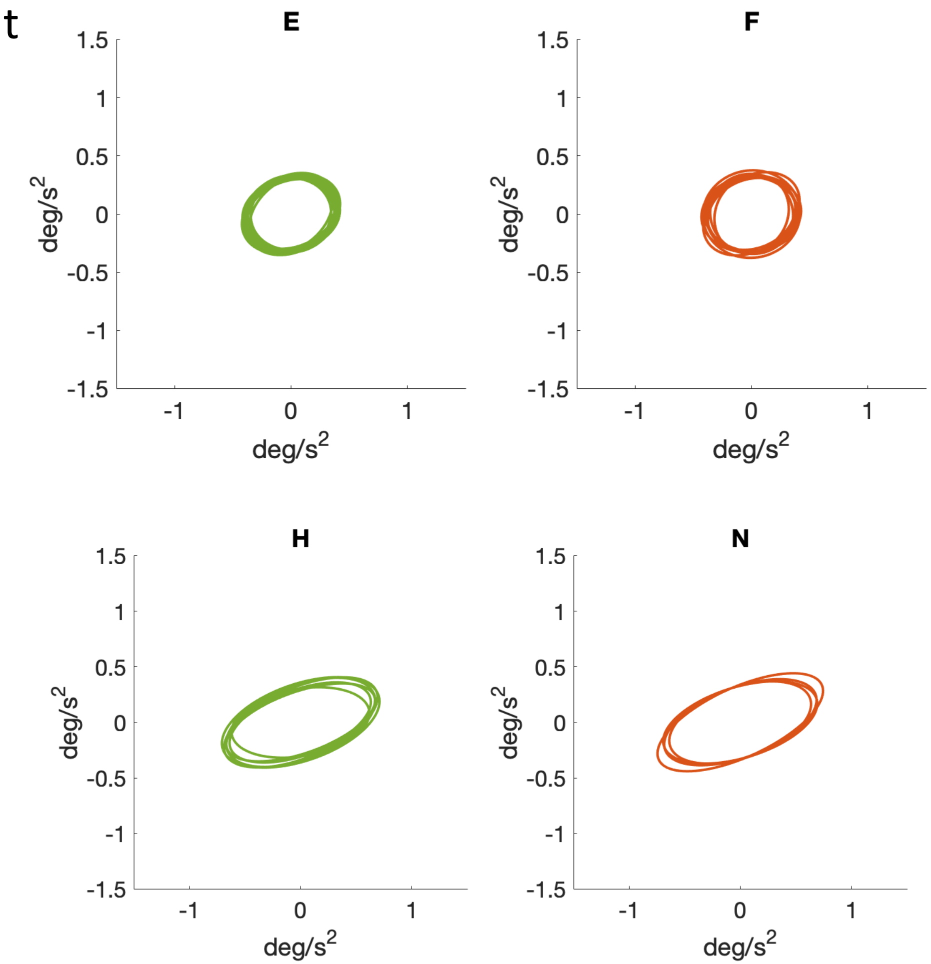




Difference in accelerations between pairs

Preliminary data suggests no directional bias between different letters within blocks, but difference between different pairs





Overview of Results from All 3 Tasks

	Grating detection	Grating discrimination	Letter discrimination
Between Stimuli	No difference	Maybe?	No difference
Between Stimuli – trial-by-trial	No difference	No difference	No difference
Between Tasks – trial-by-trial	N.A.	N.A.	YES (need more data)

Current Hypotheses & Next Steps

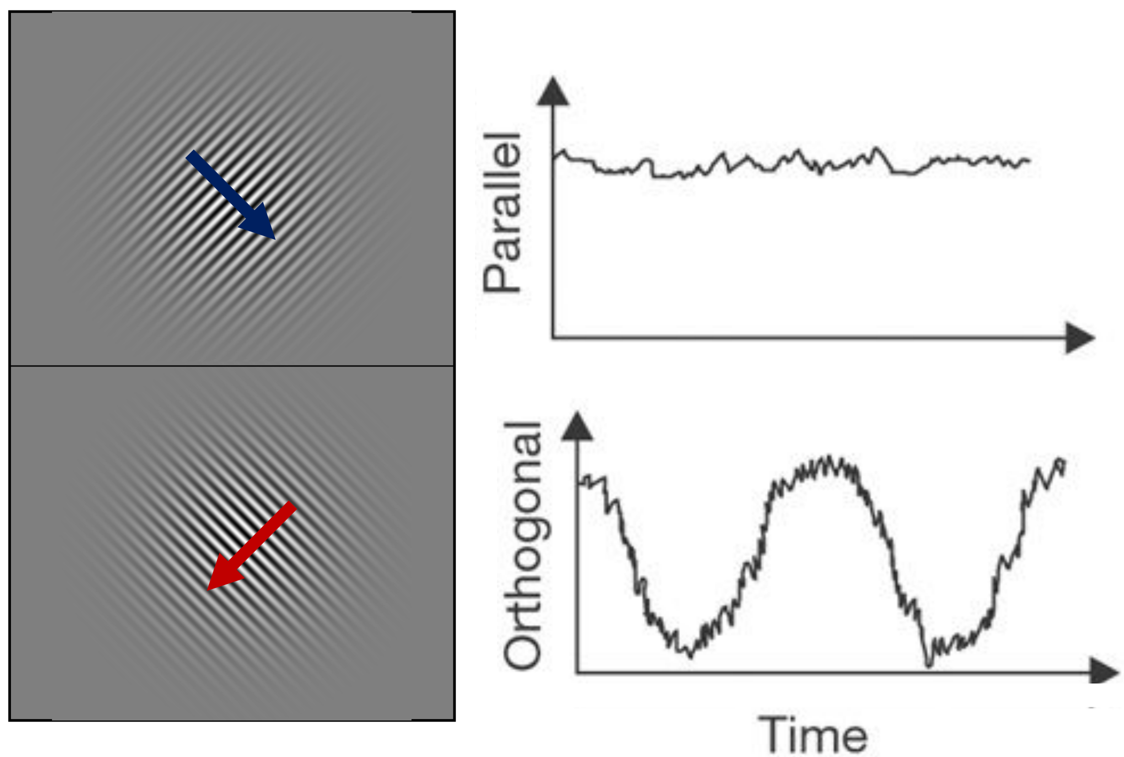
Switch Speaker

Optimal bias ocular drifts strategy
– from the luminance change point of view

STIMULUS-DRIVEN
increase luminance power
for all stimuli

Active, controlled anisotropic FEM

FEM would change
for each stimulus

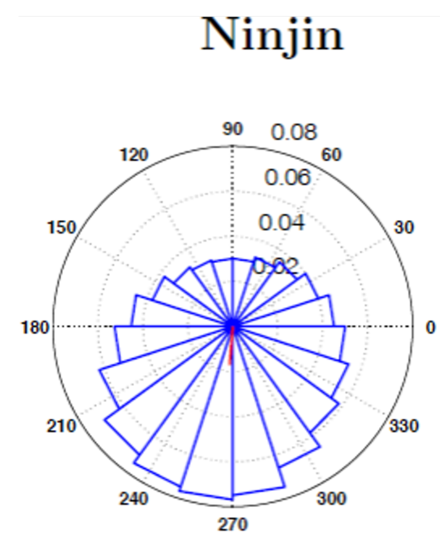


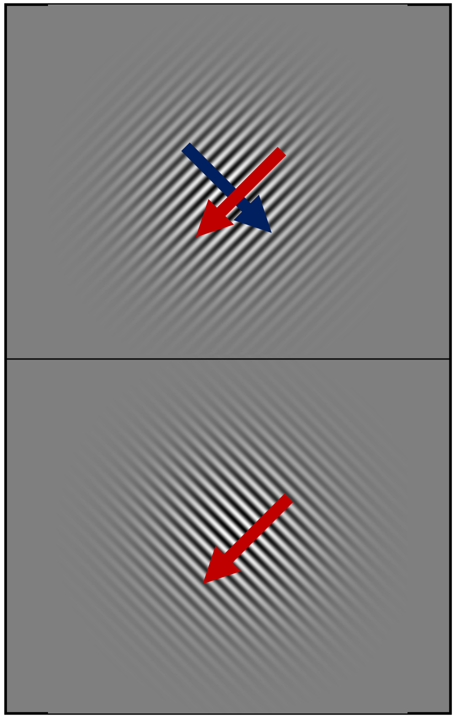
Optimal bias ocular drifts strategy
– from the luminance change point of view

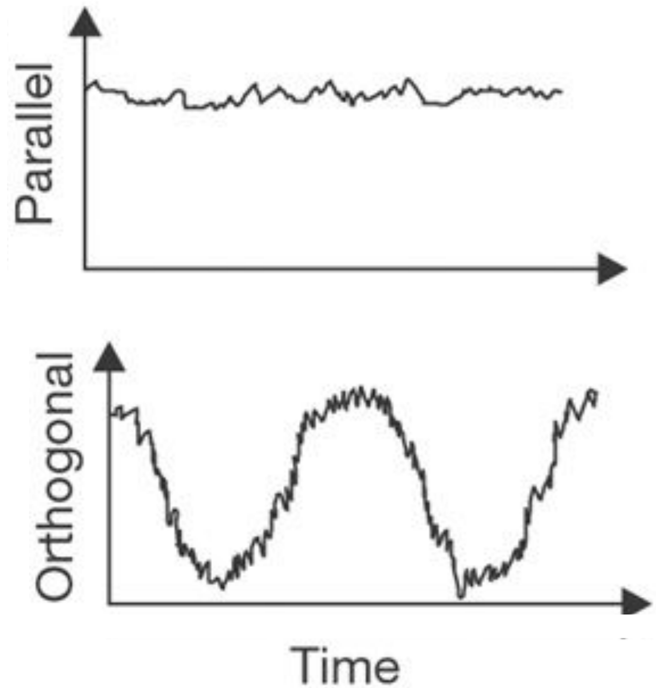
STIMULUS-DRIVEN
increase luminance power
for all stimuli

Active, … rolled aniso… ic FEM

FEM … ge
f… ch stim…



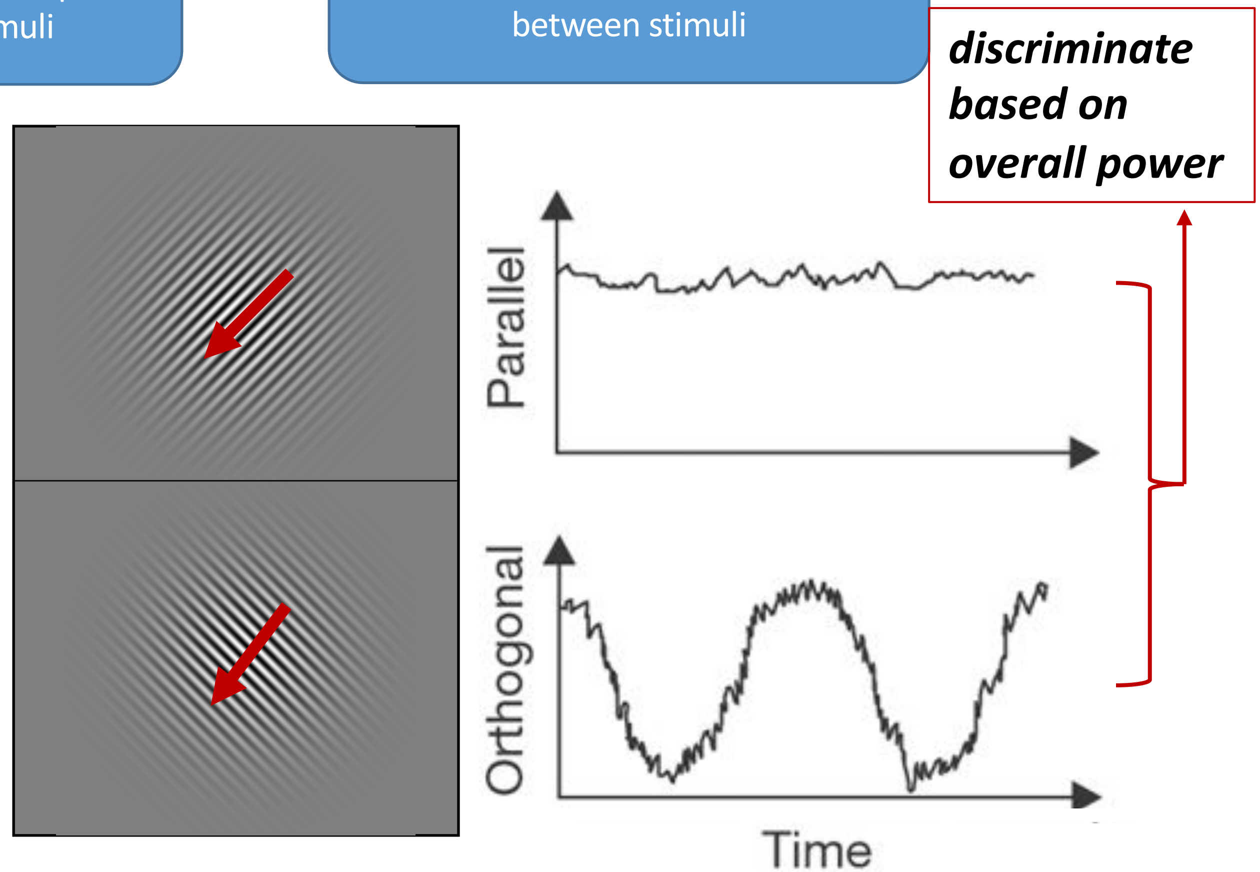




Optimal bias ocular drifts strategy
– from the luminance change point of view

STIMULUS-DRIVEN
increase luminance power
for all stimuli

TASK-DRIVEN
increase luminance power difference
between stimuli



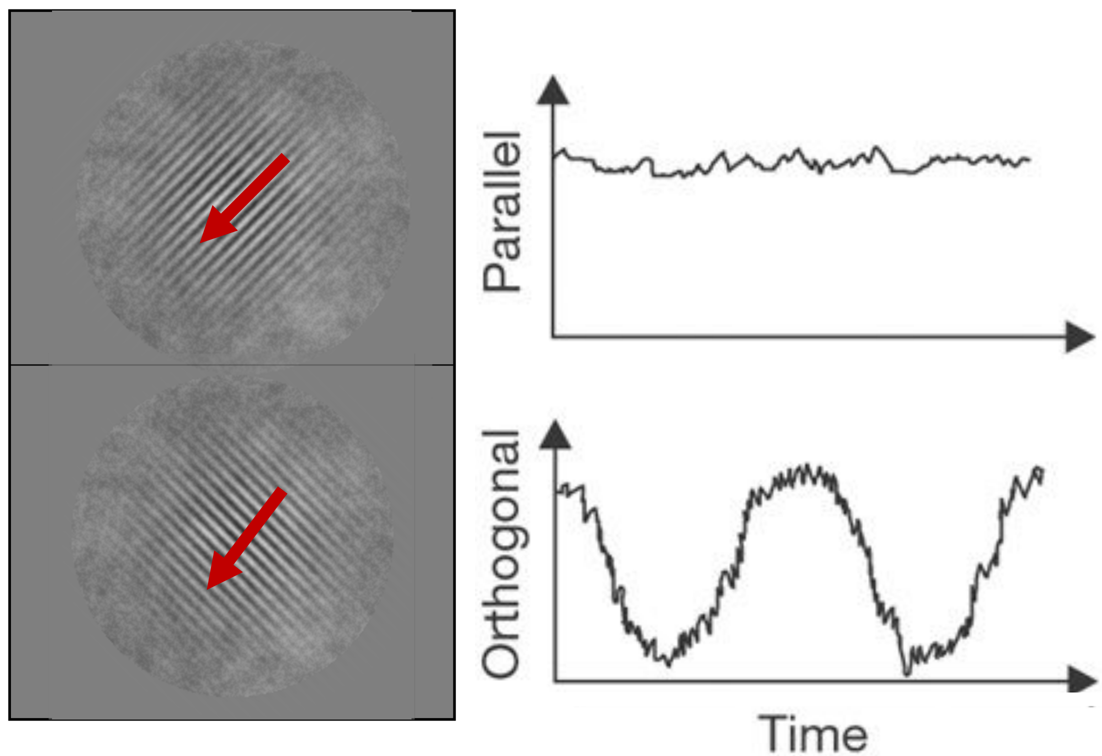
Optimal bias ocular drifts strategy
– from the luminance change point of view

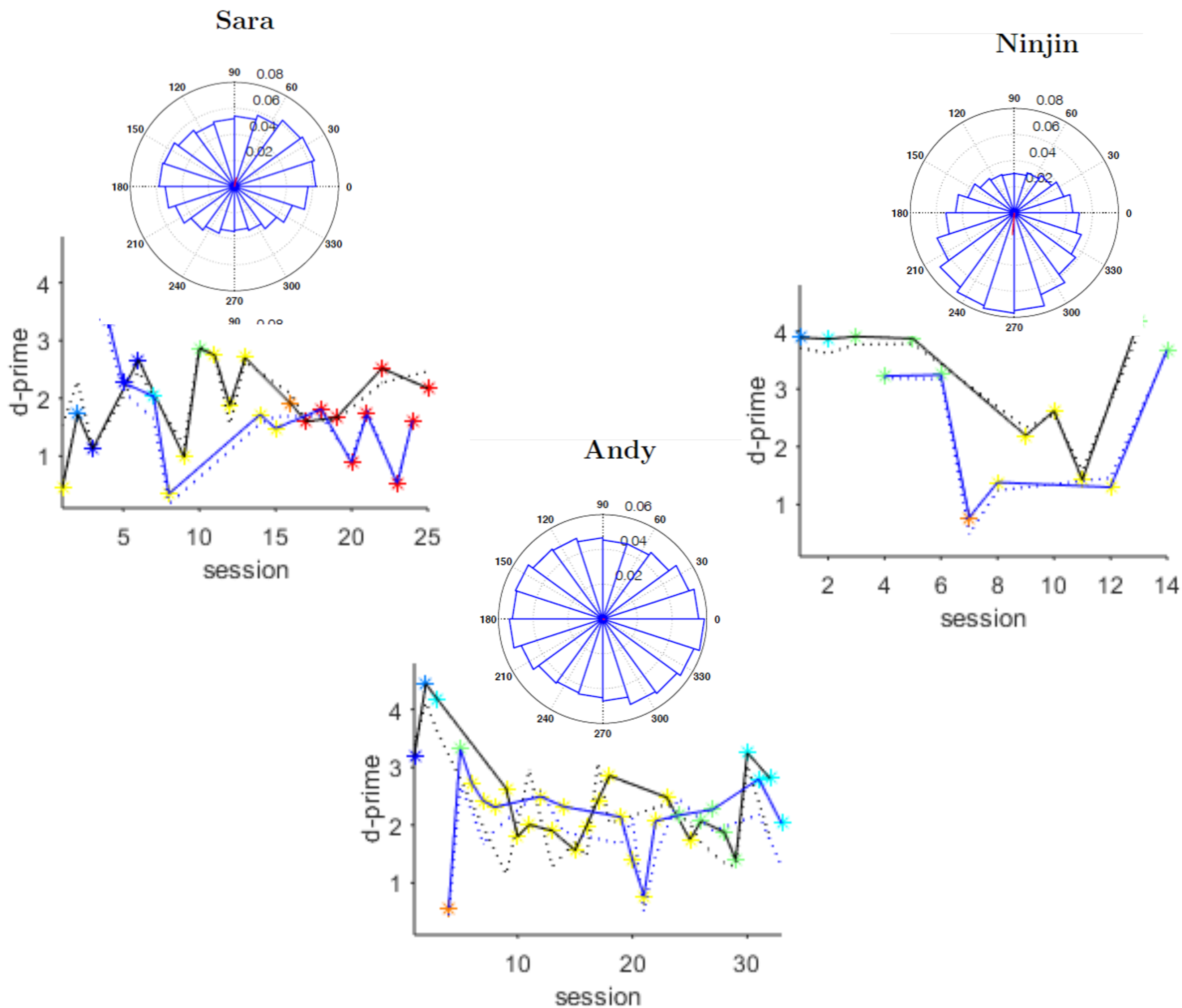
STIMULUS-DRIVEN
increase luminance power
for all stimuli

TASK-DRIVEN
increase luminance power difference
between stimuli

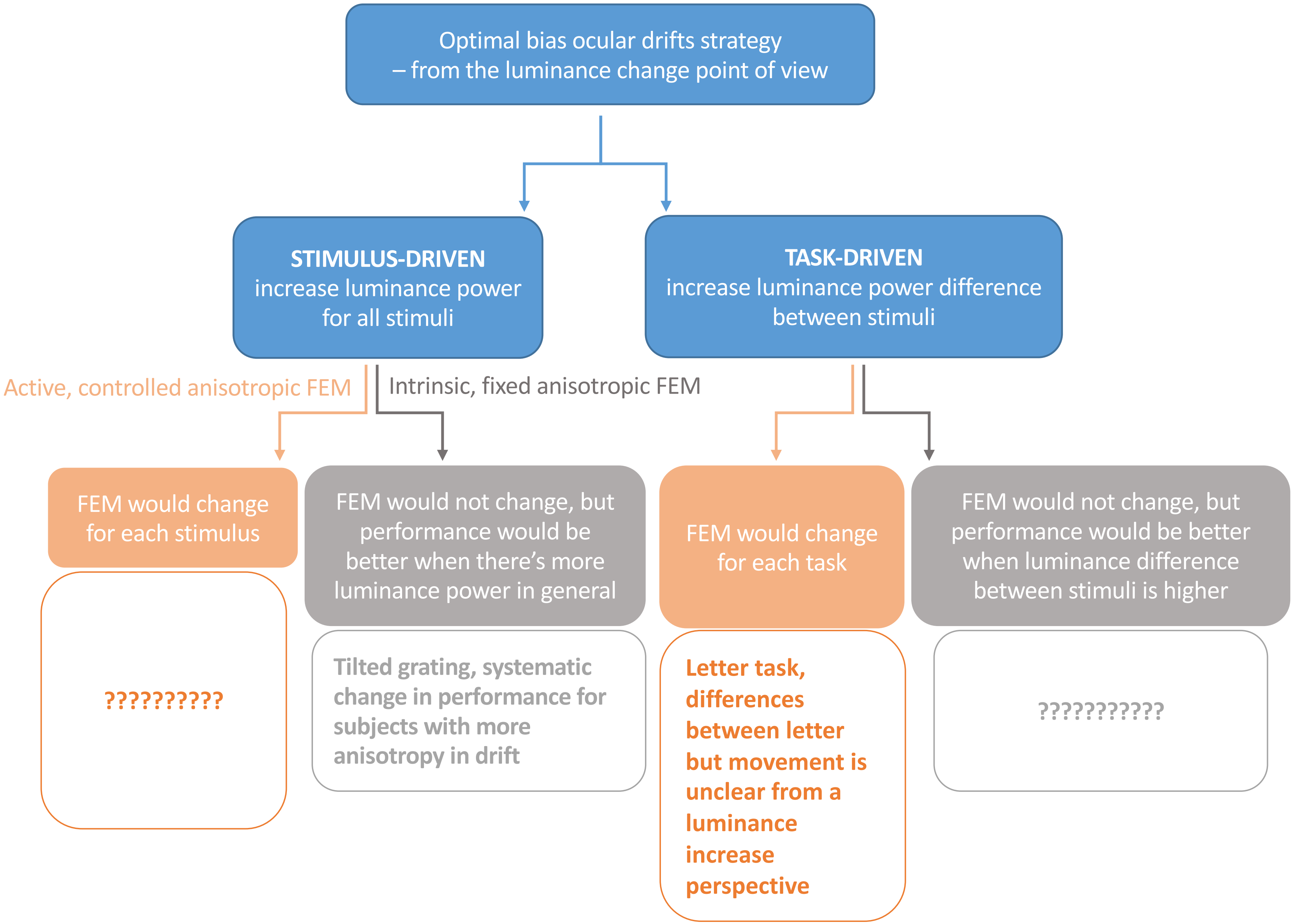
Detection in noise?

Most benefit when eye movement is orthogonal





Sara
Ninjin
Andy
d-prime
session


Optimal bias ocular drifts strategy
– from the luminance change point of view
STIMULUS-DRIVEN
increase luminance power
for all stimuli
TASK-DRIVEN
increase luminance power difference
between stimuli
Active, controlled anisotropic FEM
Intrinsic, fixed anisotropic FEM
FEM would change
for each stimulus
??????????
FEM would not change, but
performance would be
better when there's more
luminance power in general
Tilted grating, systematic
change in performance for
subjects with more
anisotropy in drift
FEM would change
for each task
Letter task,
differences
between letter
but movement is
unclear from a
luminance
increase
perspective
FEM would not change, but
performance would be better
when luminance difference
between stimuli is higher
???????????

Next Steps

- Develop and test measures of drift anisotropy
- Analyze multiple datasets for evidence of change in drift anisotropy across conditions
- *a priori* definition of an optimal drift anisotropy for any given task and stimulus that is widely adaptable