Transfer Matrix Method

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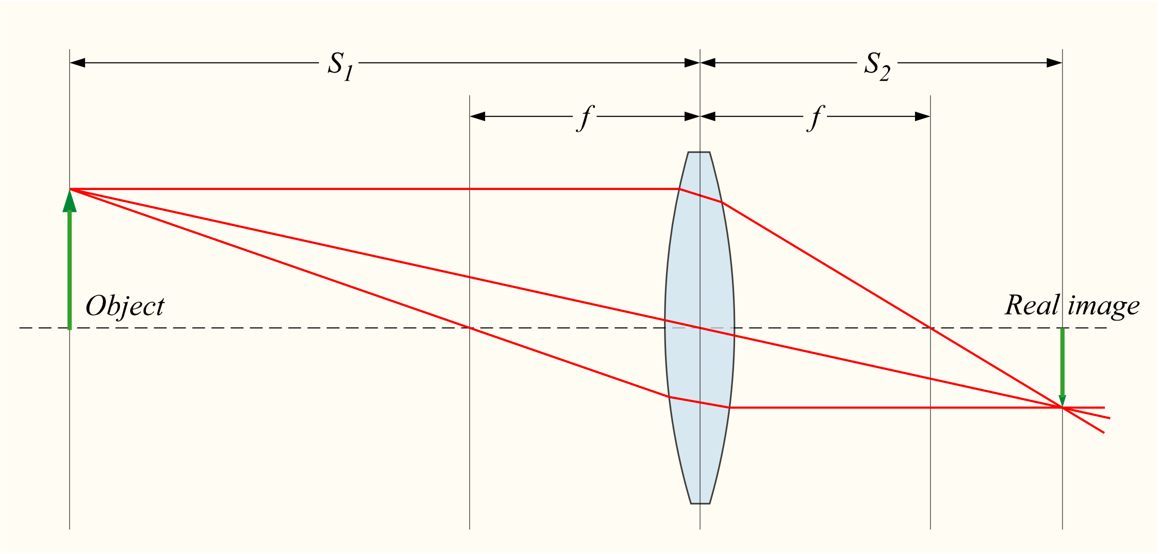
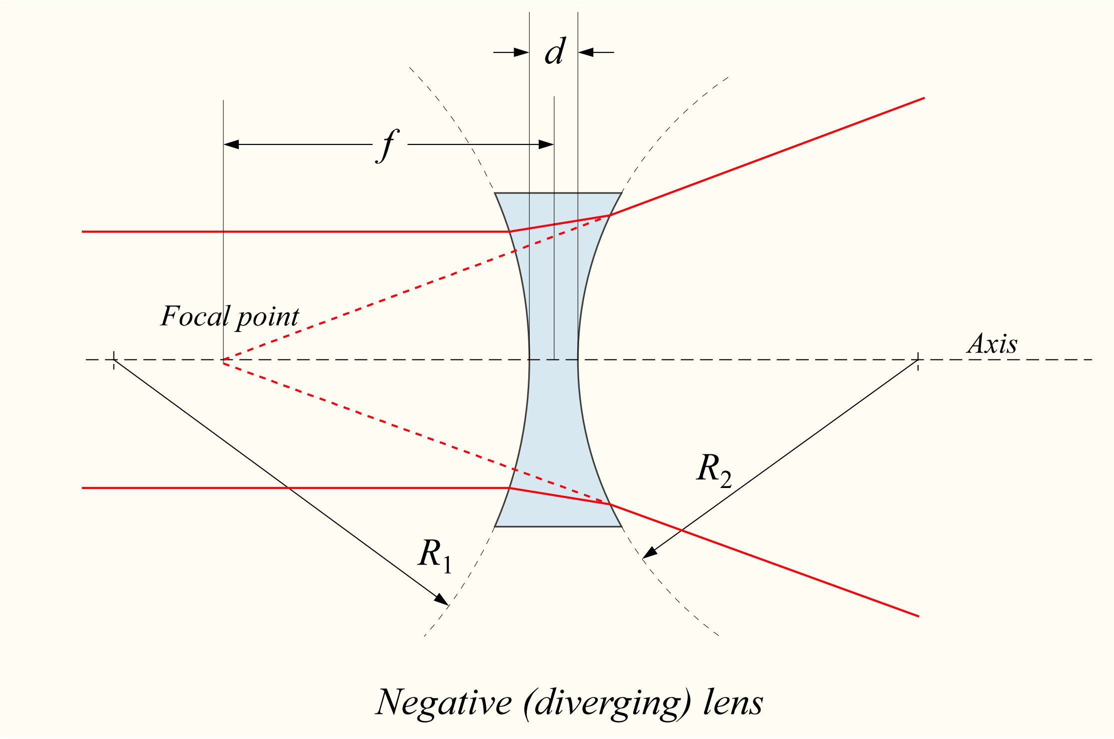
Thin lens formula

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$



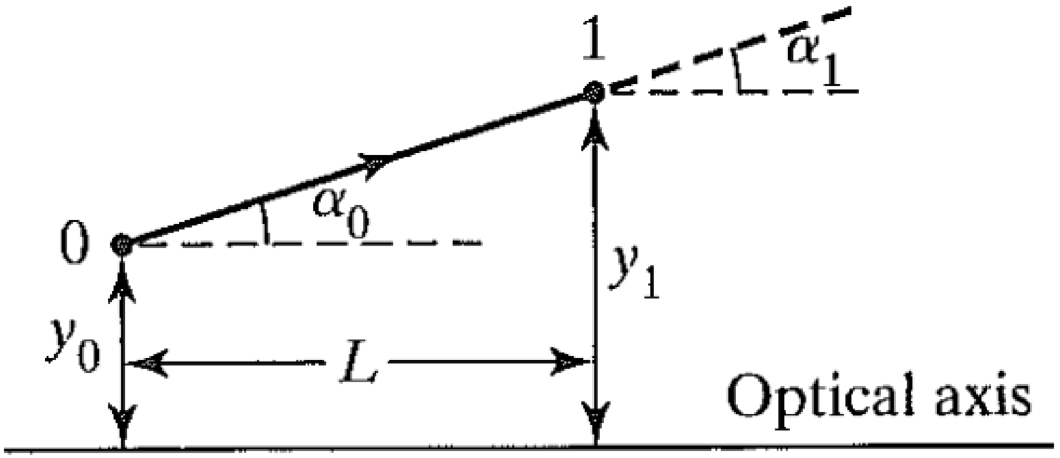
Lensmaker's equation

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$



Transfer Matrix Method – Simple translation

$$y_1 = (1)y_0 + (L)\alpha_0$$
$$\alpha_1 = (0)y_0 + (1)\alpha_0$$



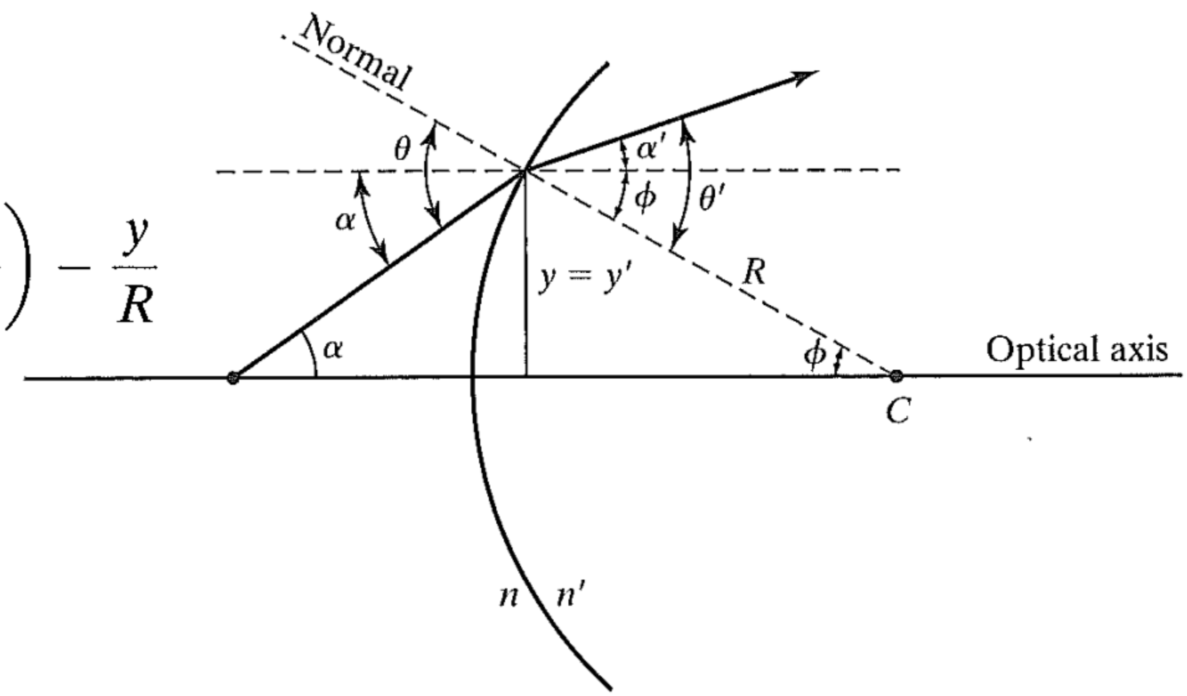
$$\begin{bmatrix} y_1 \\ \alpha_1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix}$$

Transfer Matrix Method – Refraction

$n\theta = n'\theta'$ (Snell's law)

$$\alpha' = \left(\frac{n}{n'}\right)\theta - \frac{y}{R} = \left(\frac{n}{n'}\right)\left(\alpha + \frac{y}{R}\right) - \frac{y}{R}$$

$$= \left(\frac{1}{R}\right)\left(\frac{n}{n'} - 1\right)y + \left(\frac{n}{n'}\right)\alpha$$



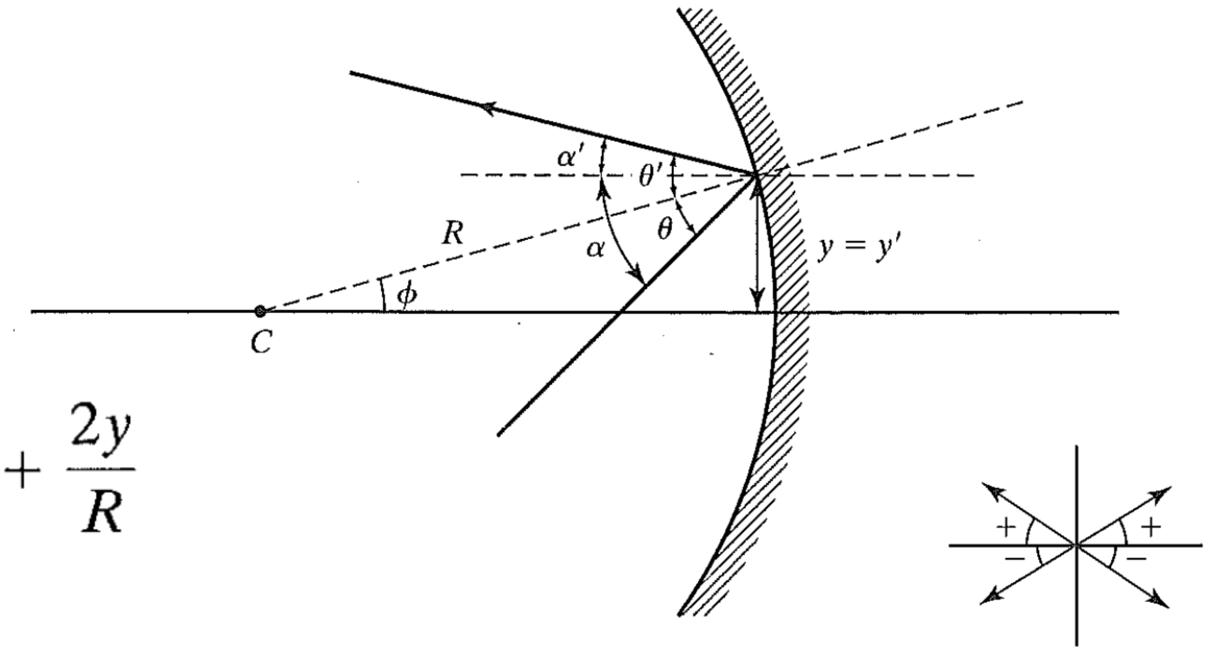
$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n}{n'} - 1\right) & \frac{n}{n'} \end{bmatrix}} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

Transfer Matrix Method – Reflection

$$\alpha = \theta + \phi = \theta + \frac{y}{-R}$$

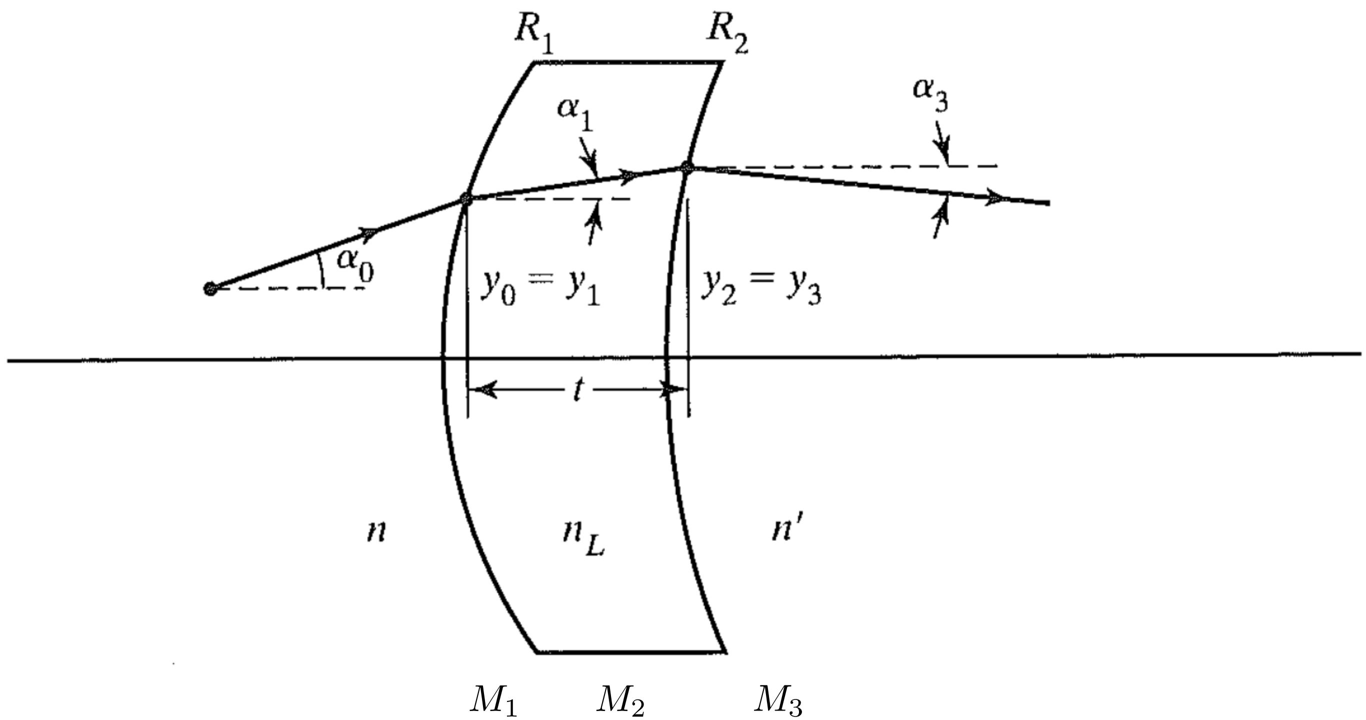
$$\alpha' = \theta' - \phi = \theta' - \frac{y}{-R}$$

$$= \theta' + \frac{y}{R} = \theta + \frac{y}{R} = \alpha + \frac{2y}{R}$$



$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

Transfer Matrix Method – Thin Lens



$$M = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{n_L - n'}{n' R_2} & \frac{n_L}{n'} \end{bmatrix}}_{M_3} \underbrace{\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}}_{M_2} \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{n - n_L}{n_L R_1} & \frac{n}{n_L} \end{bmatrix}}_{M_1}$$

$$t \to 0 \ = \begin{bmatrix} 1 & 0 \\ \frac{n_L - n}{n R_2} & \frac{n_L}{n} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{n - n_L}{n_L R_1} & \frac{n}{n_L} \end{bmatrix}$$

Transfer Matrix Method – Thin Lens

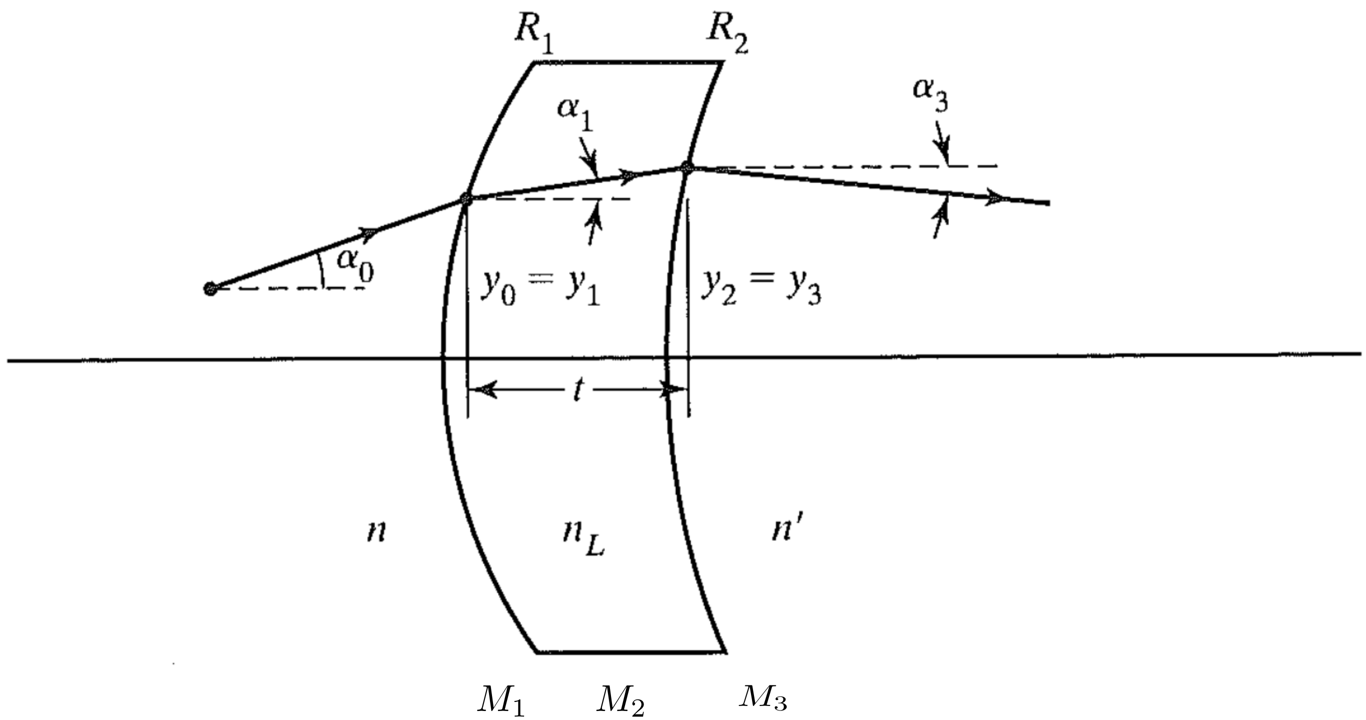
$$M = \begin{bmatrix} 1 & 0 \\ \dfrac{n_L - n}{n}\left(\dfrac{1}{R_2} - \dfrac{1}{R_1}\right) & 1 \end{bmatrix}$$

$$\frac{1}{f} = \frac{n_L - n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thin Les Formula

$$M = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

Transfer Matrix Method – Thick Lens



$$M = \underset{M_3}{\begin{bmatrix} 1 & 0 \\ \dfrac{n_L - n'}{n' R_2} & \dfrac{n_L}{n'} \end{bmatrix}} \underset{M_2}{\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}} \underset{M_1}{\begin{bmatrix} 1 & 0 \\ \dfrac{n - n_L}{n_L R_1} & \dfrac{n}{n_L} \end{bmatrix}}$$

$$n = n' \qquad = \begin{bmatrix} 1 + \dfrac{(n - n_L)t}{n_L R_1} & \dfrac{nt}{n_L} \\ -(n_L - n)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2} + \dfrac{(n_L - n)t}{n R_1 R_2}\right] & \dfrac{n_L R_2 + (n_L - n)t}{R_2} \end{bmatrix}$$