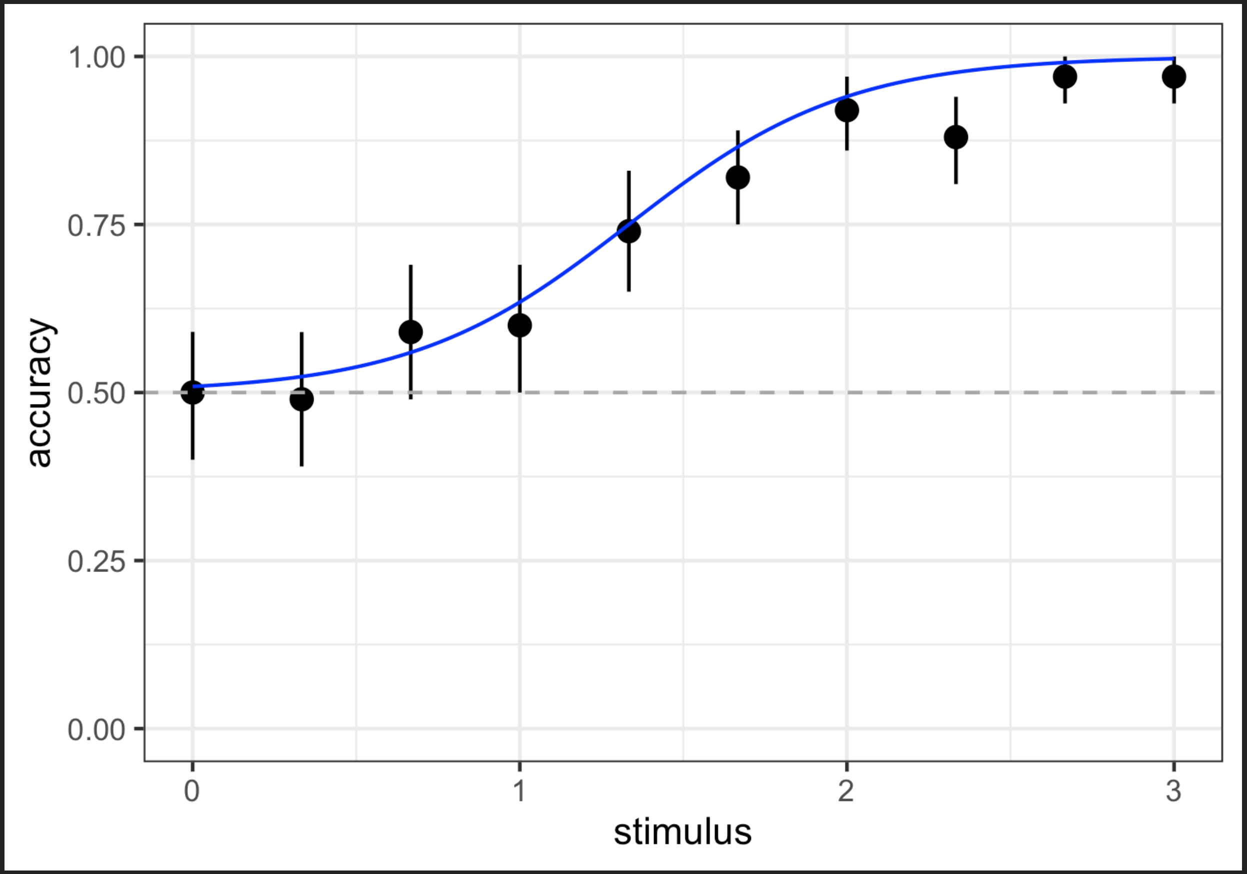
THE PERILS OF (NOT) TAKING INTO ACCOUNT LAPSING IN THE ESTIMATION OF PSYCHOMETRIC FUNCTIONS

MIND THE LAPSE

PSYCHOMETRIC MODELS

- Linking the observed *stimulus* to the observed *response*



USE

- To study effects of stimuli on perception
- To study whether this link (the psychometric function) is affected by a manipulation of interest.
  - Differences across populations
  - Differences in the perceptual context
  - etc.

COMMON APPROACH

- Fit psychometric model to **different groups** of participant (ASD vs. NT) or **different conditions** of same participants.
- Obtain parameterization of psychometric function (**threshold, slope**, …)
- Submit these parameter estimates to secondary analyses

- This **throws away information** (and does not optimally discount for uncertainty) but won't get into that today.

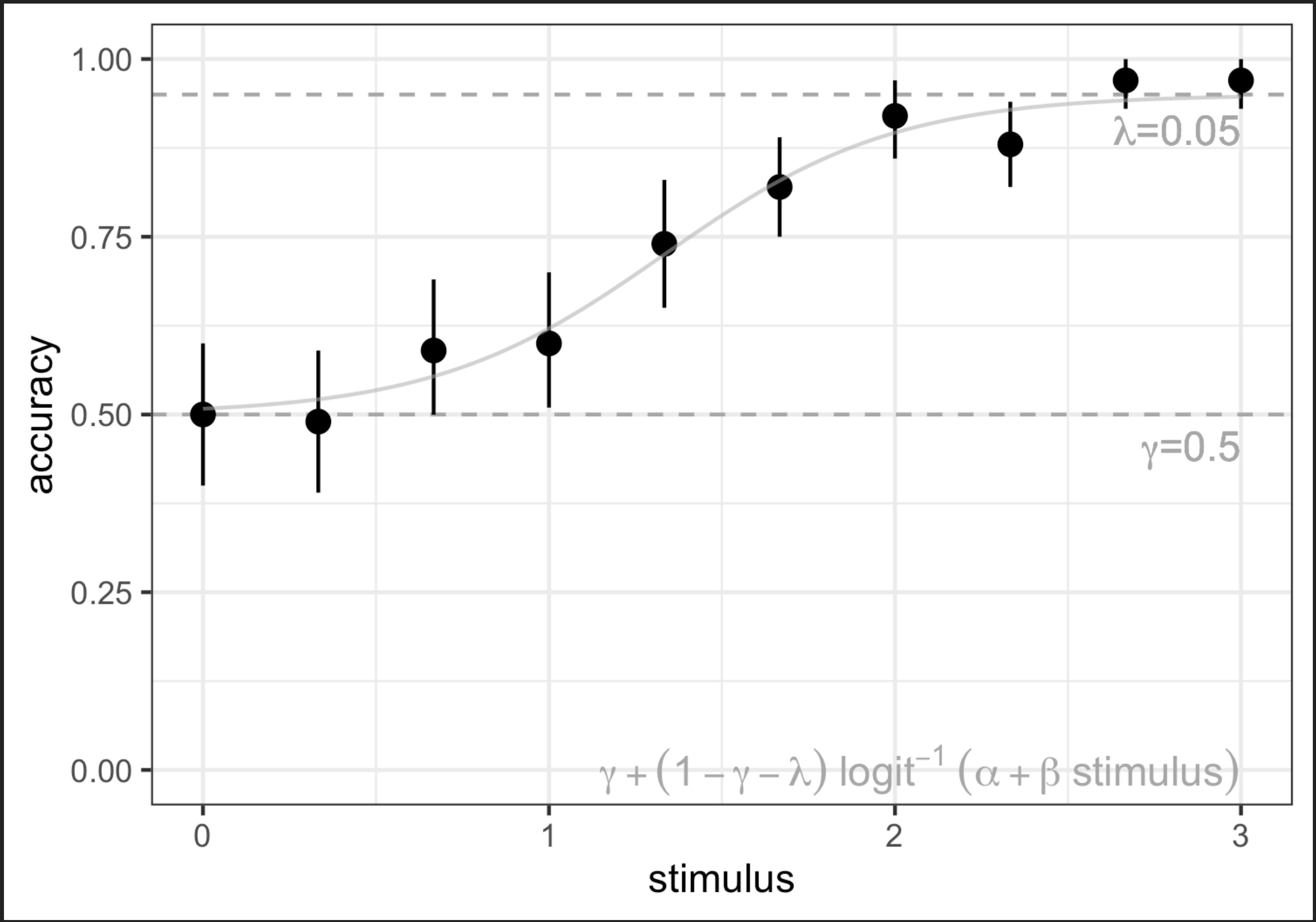
BUT

- Psychometric models have two parts:

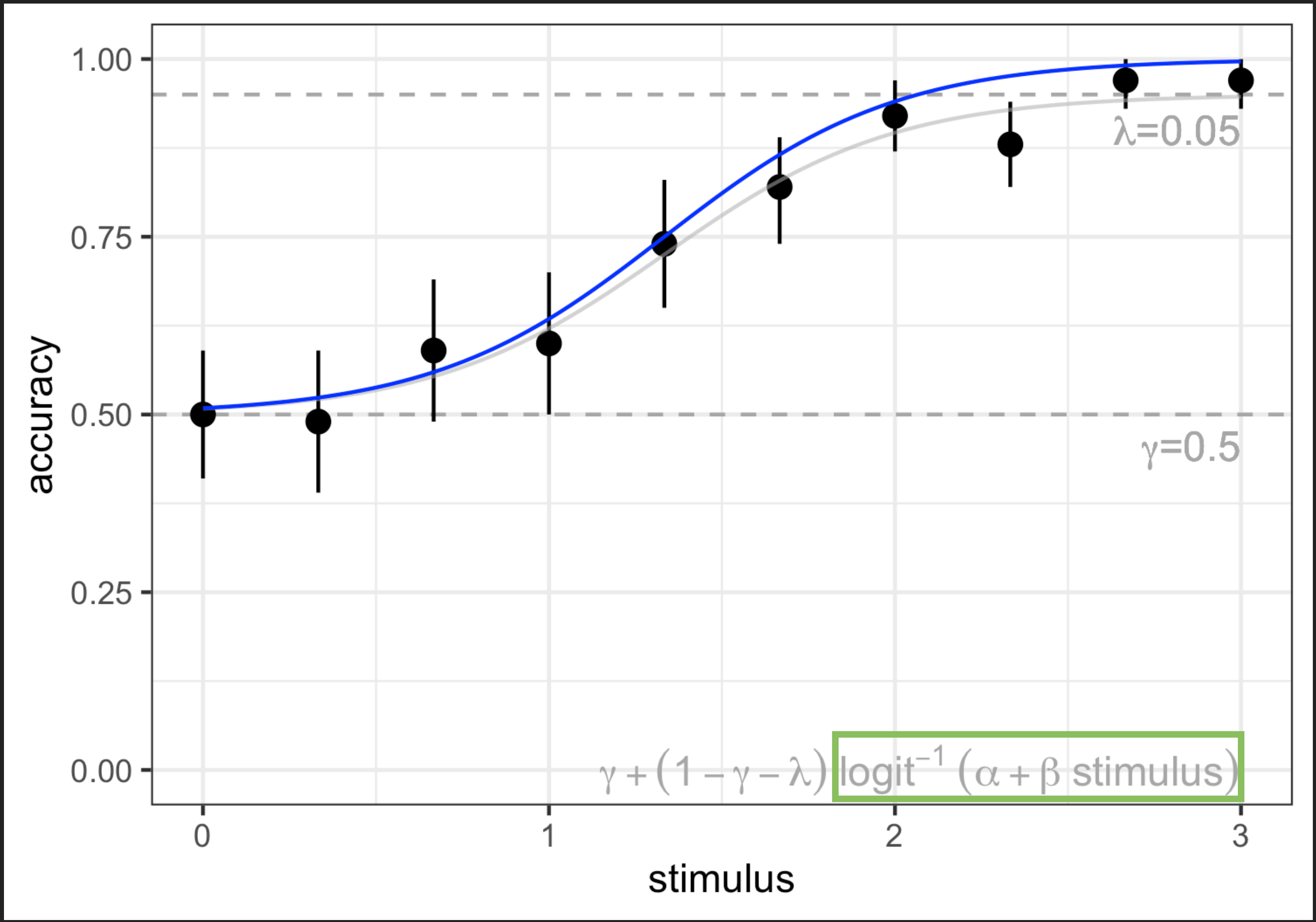
$$\psi(x;\alpha,\beta,\gamma,\lambda) = \boxed{\gamma + (1-\gamma-\lambda)}\boxed{F_G(x;\alpha,\beta)}. \qquad (1b)$$

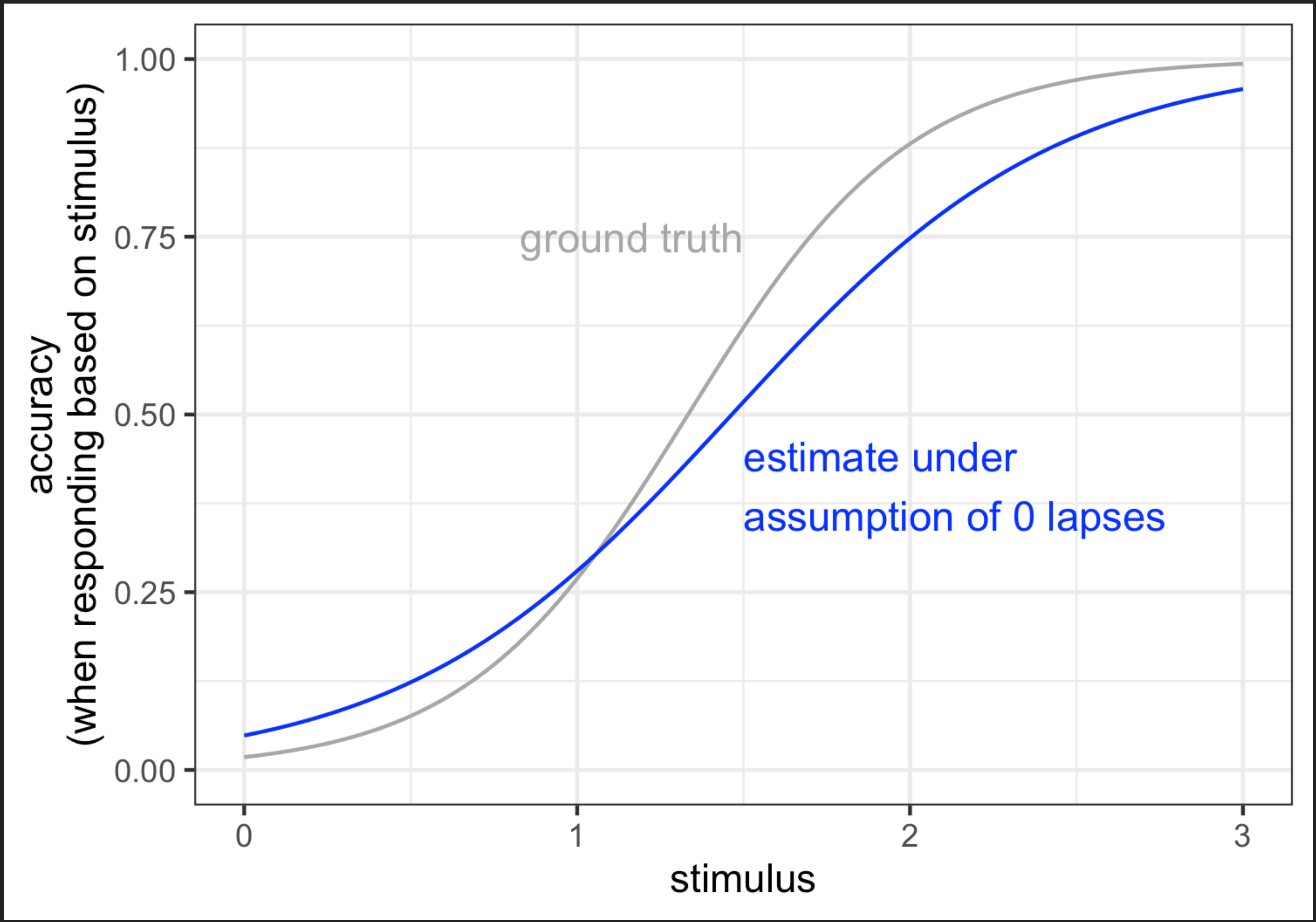
- **Perceptual model:** effect of stimulus
  - Threshold
  - Slope
- **Lapsing model:** proportion of trials on which subject does not respond based on stimulus *x* (and what happens on those trials) giving lower (chance γ) and upper bound $(1-\lambda)$ performance

MIND THE LAPSE (λ)



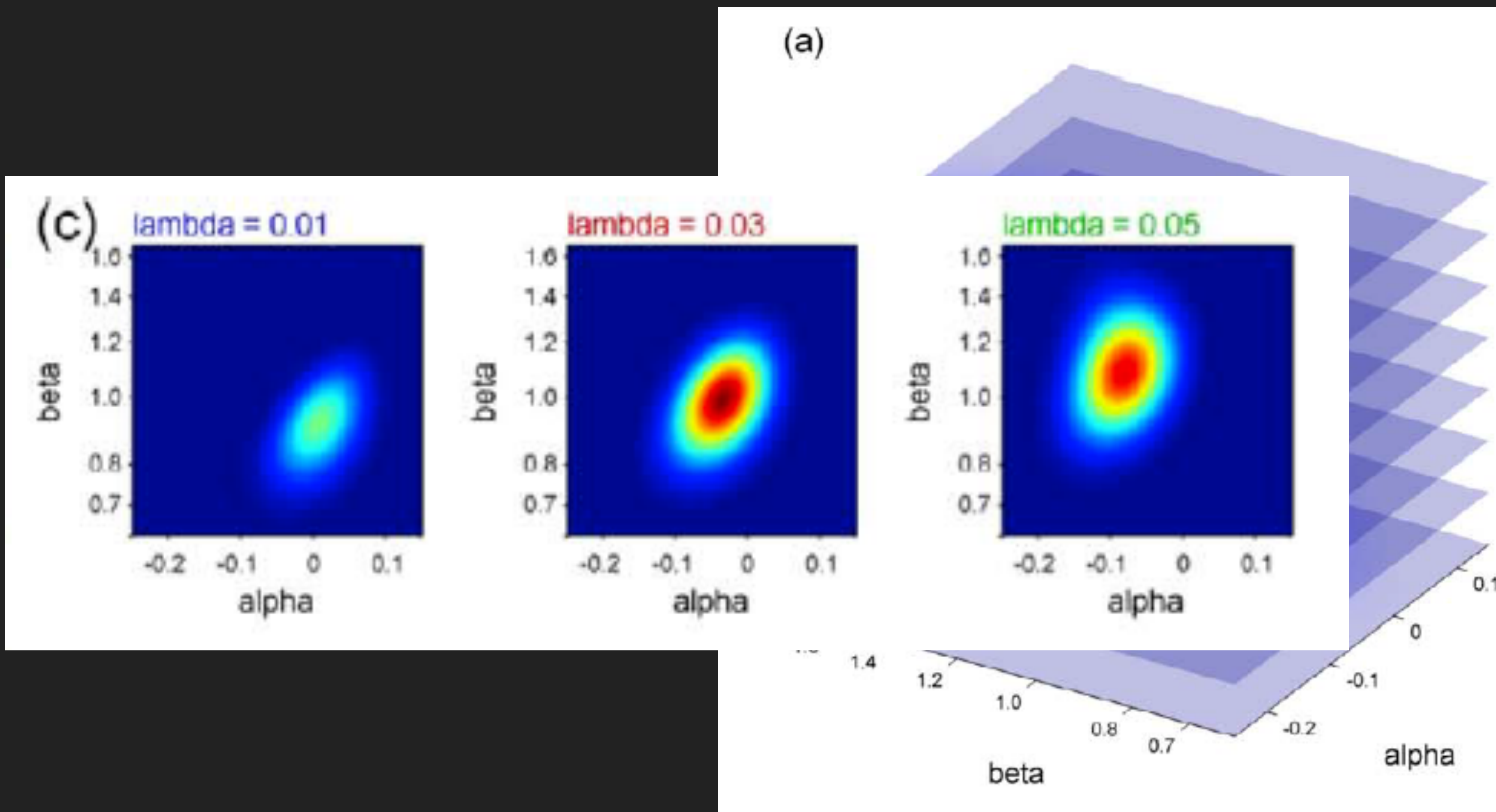
MIND THE LAPSE (λ)





accuracy
(when responding based on stimulus)
1.00
0.75
0.50
0.25
0.00
0
1
2
3
stimulus
ground truth
estimate under
assumption of 0 lapses

CORRELATIONS AMONG THE 3 PARAMETERS



[from Prins 2013]

SOLUTIONS

- **Ignore nuisance parameters:** Only estimate the parameters of interest (typically, threshold and slope)

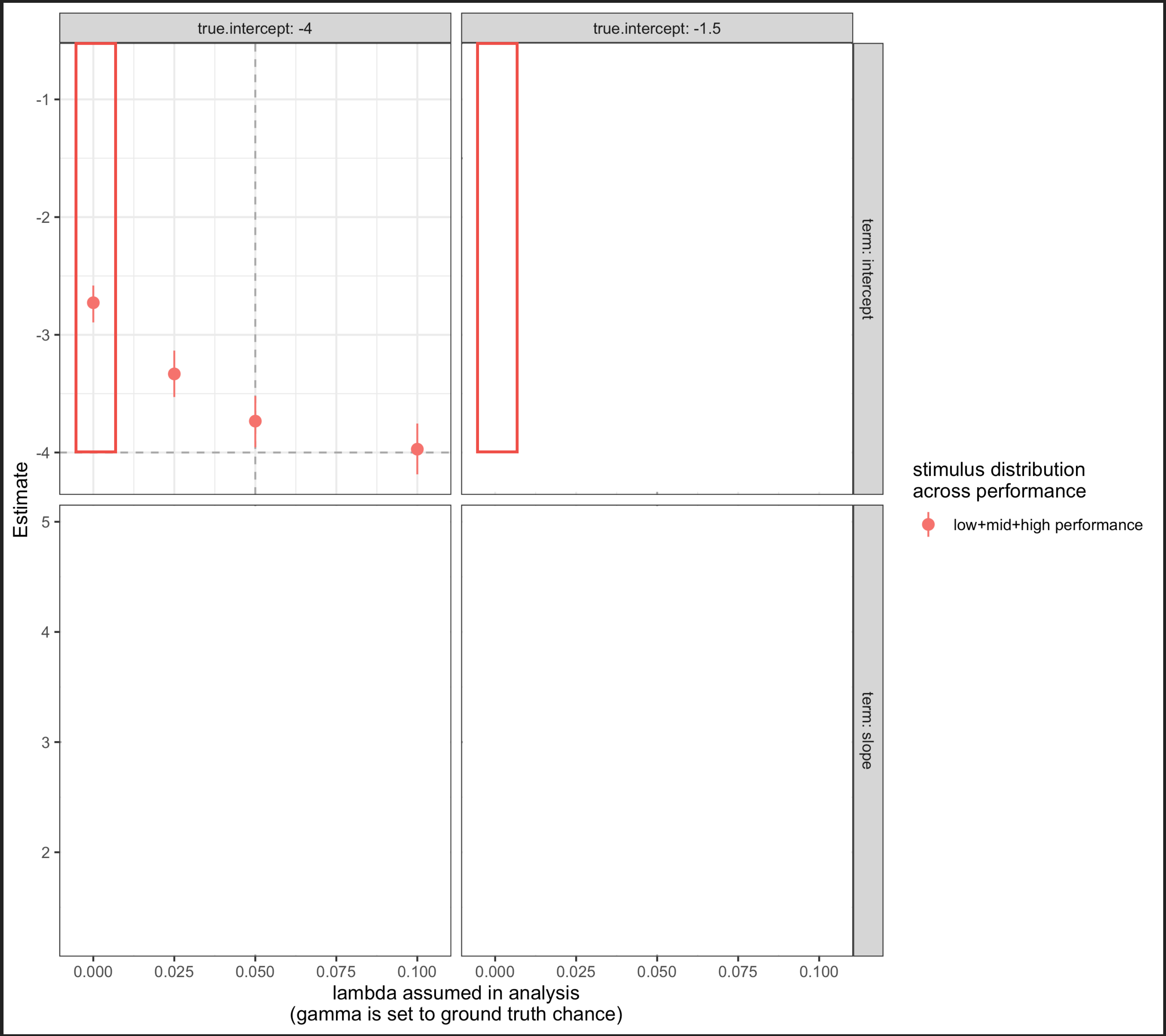
  - While ignoring lapse rate (i.e., assuming its zero)
  - Fixing lapse rate to some value by previous work

- Fix lapse rate based on **separately estimated ceiling on the same data** for (typically small n of) extreme values

SIMULATION STUDY

$$\psi(x;\alpha,\beta,\gamma,\lambda) = \boxed{\gamma + (1-\gamma-\lambda)}\boxed{F_G(x;\alpha,\beta)} \quad (1b)$$

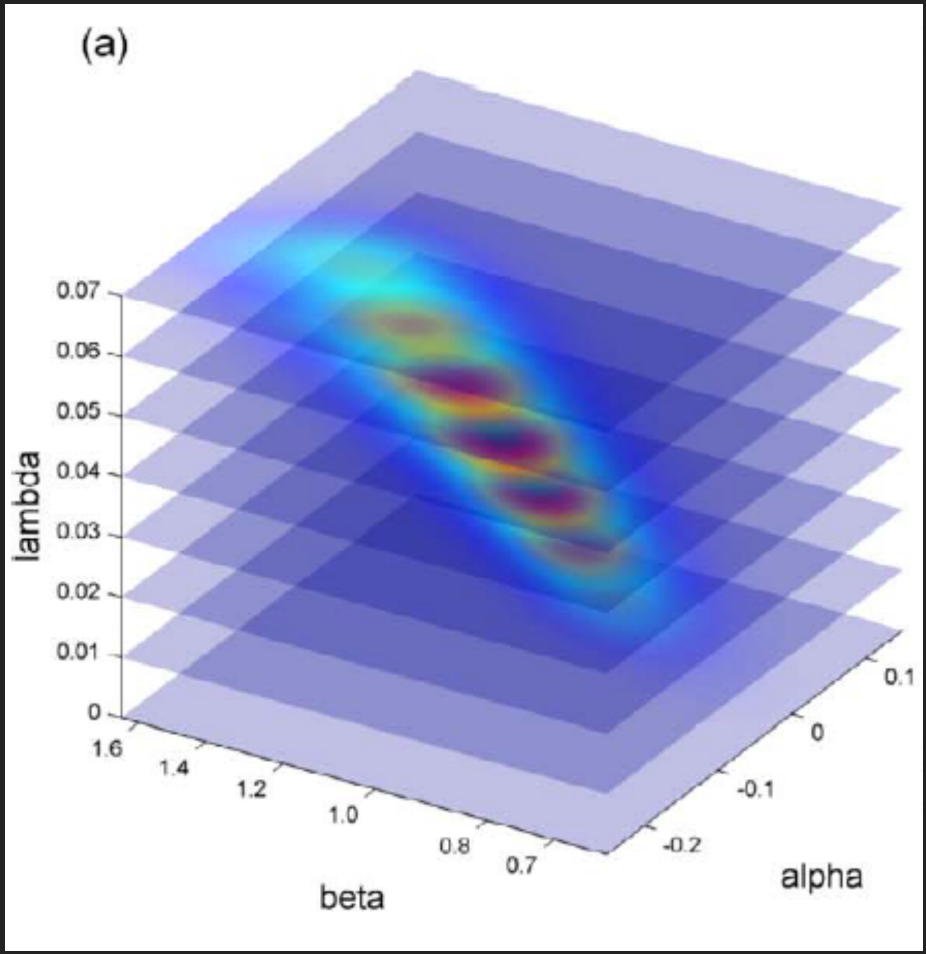
- **Generate data from ground truth**
  - Lapsing (λ=.05) psychometric model for 2 AFC (γ = .5); logistic perceptual model.
  - Considered different intercepts α and slopes β; different stimulus regimes
- **Estimate intercept $\hat{\alpha}$ and slope $\hat{\beta}$ under different assumptions about λ** (γ always correctly assumed to be chance level)
  - Sampling-fitted Bayesian model with weakly regularizing priors
- **Compare estimate to ground truth** → bias.



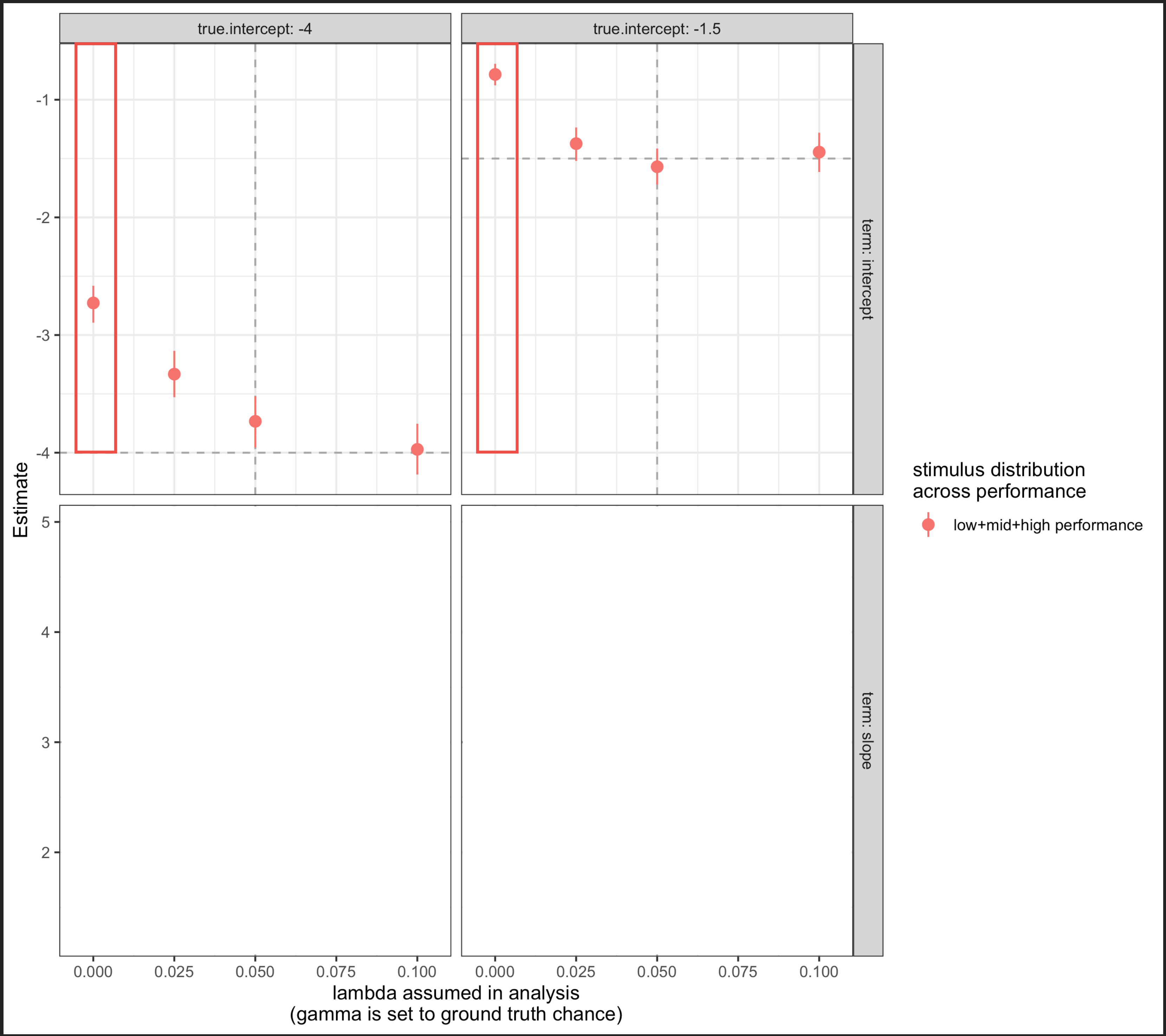
[https://github.com/tfjaeger/Tutorial-GLMM]

CORRELATIONS AMONG THE 3 PARAMETERS

- **Different assumed lapse rate** (λ) $\rightarrow$ different estimates of intercept $\hat{\alpha}$ and slope $\hat{\beta}$ (and thus also threshold stimulus)
- **Even if λ is held constant** bias is *not* constant across conditions (as it depends on true α and β)



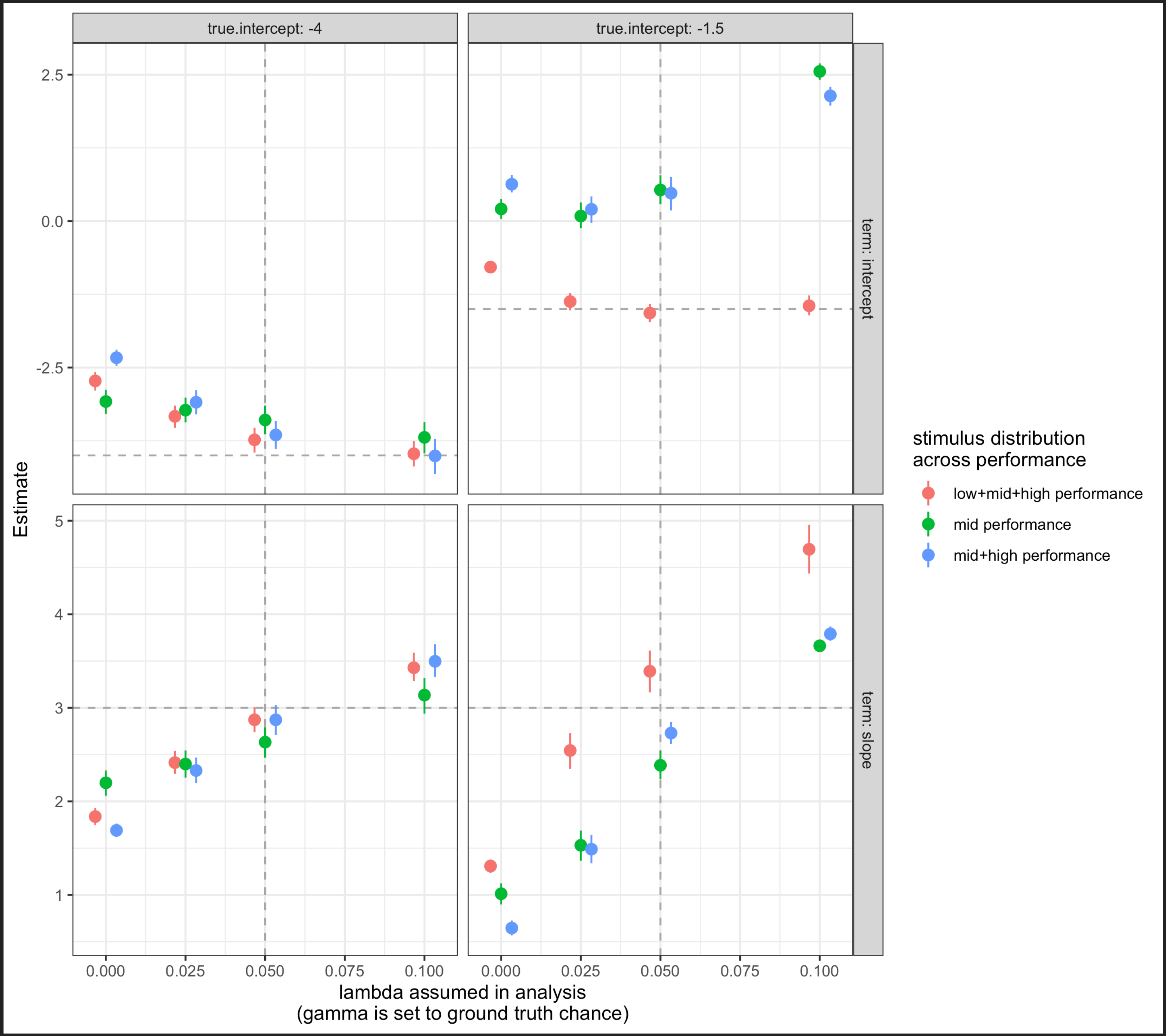
[from Prins 2013]



[https://github.com/tfjaeger/Tutorial-GLMM]

'SMART' STIMULUS SELECTION?

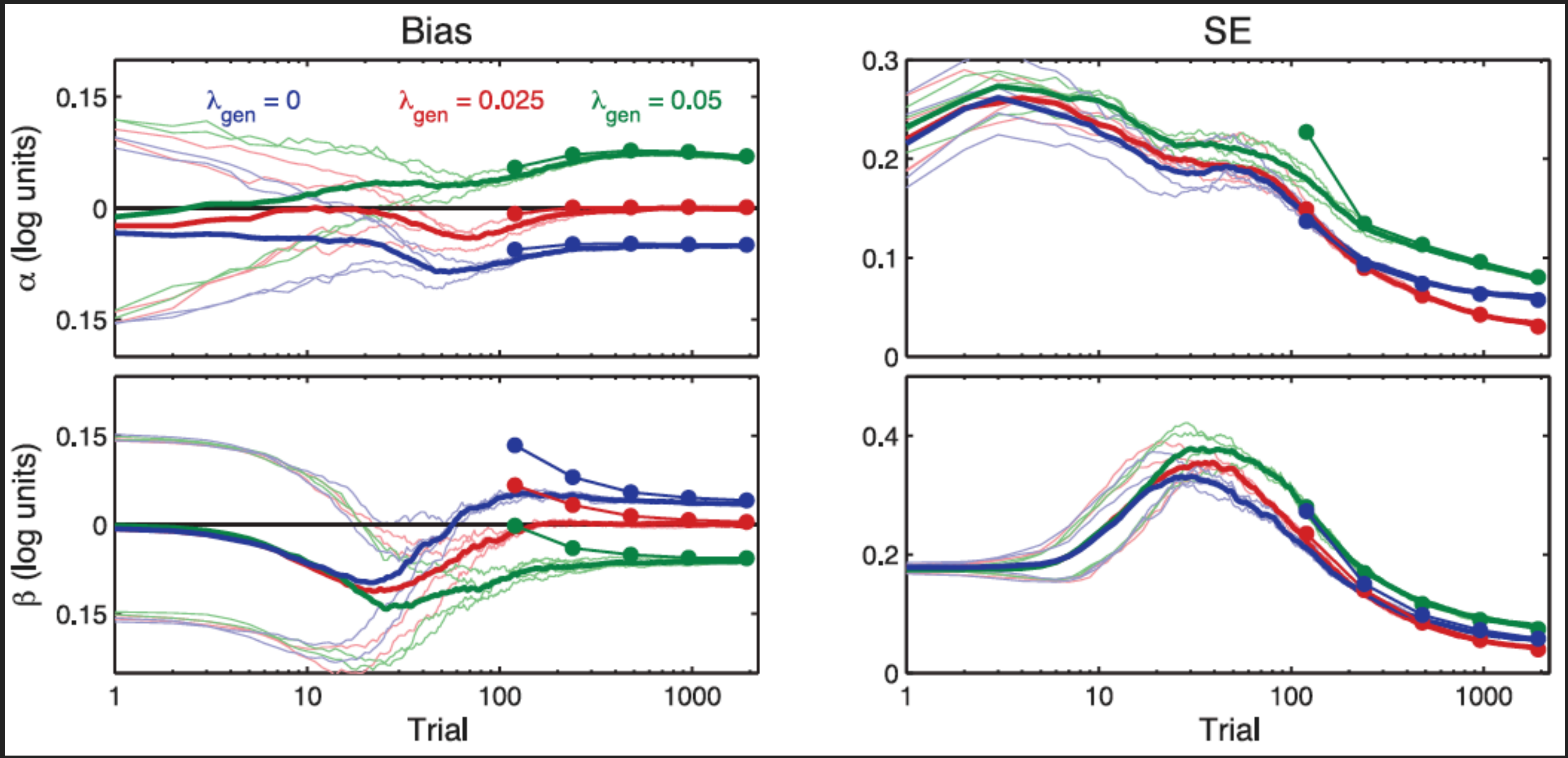
- **Common approach**: select stimuli to be maximally informative about participants' psychometric function (staircases; 'psi' method)
- **But:** informative about *what*? Typically optimized for **slope** or **threshold**.
- This is great … as long as the assumptions under which data is elicited are matched by the data. If not **this may make things worse!** (Prins, 2013)



[https://github.com/tfjaeger/Tutorial-GLMM]

COLLECT DATA THROUGH PSI-METHOD

- **Bias and uncertainty in estimates of threshold and slope** for three different ground truth λ**s** (green, red, blue) if fit under assumption of λ = .03.



[from Prins 2013]

PROBLEM REDUCED BUT REMAINS WHEN λ IS ESTIMATED

- Data collected in mid-performance range (psi-method) → **hard to estimate λ!**

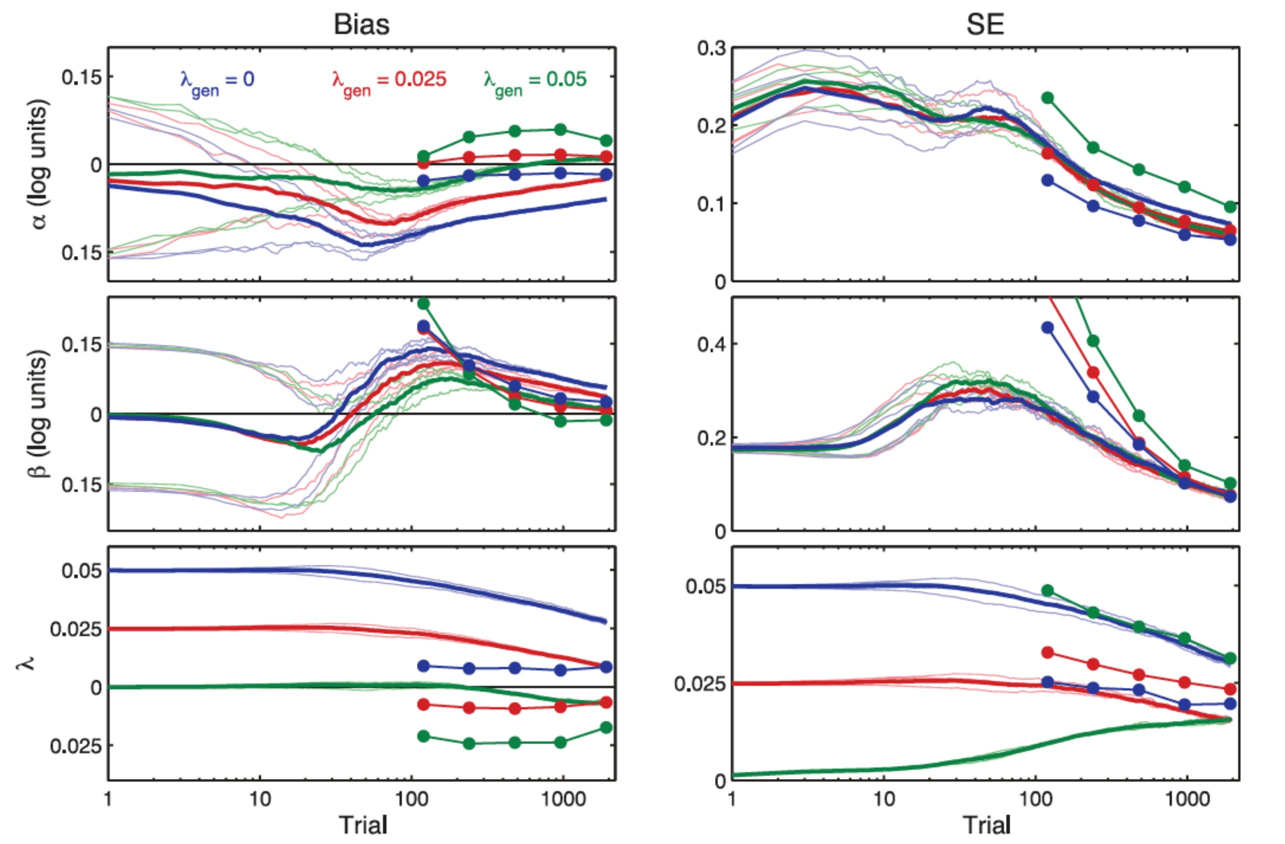


Figure 6. Bias and standard error of Bayesian (lines) and ML (symbols) parameter estimates when the data from Figure 3 are refitted while the lapse rate is allowed to vary.

[from Prins 2013]

WHEN λ IS ASSUMED

- Even with almost 2000 trials, **threshold and slopes estimates remain correlated** (generating λ = .025; assumed λ = .03)

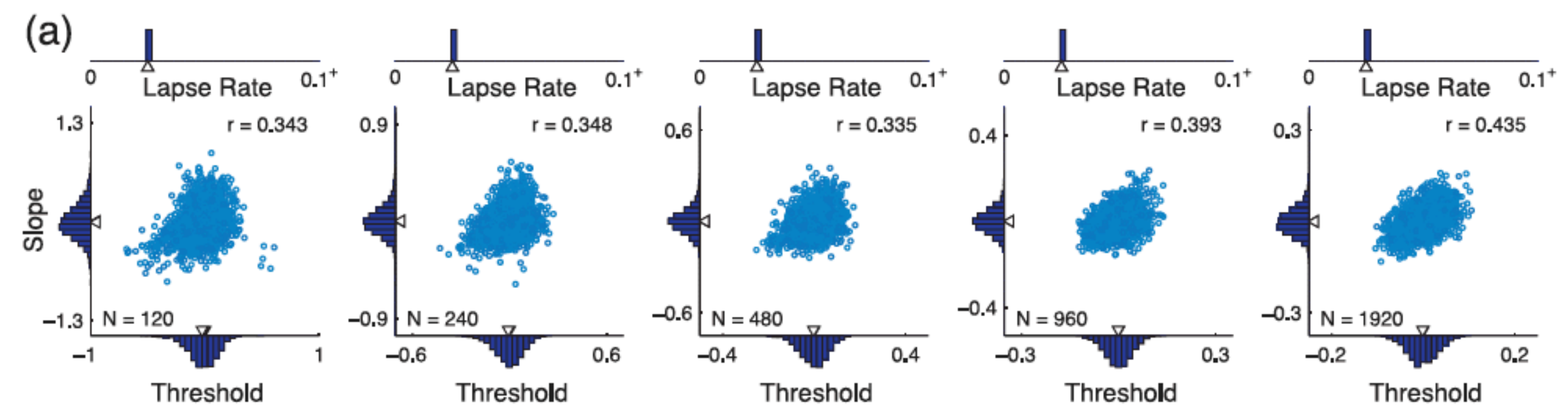
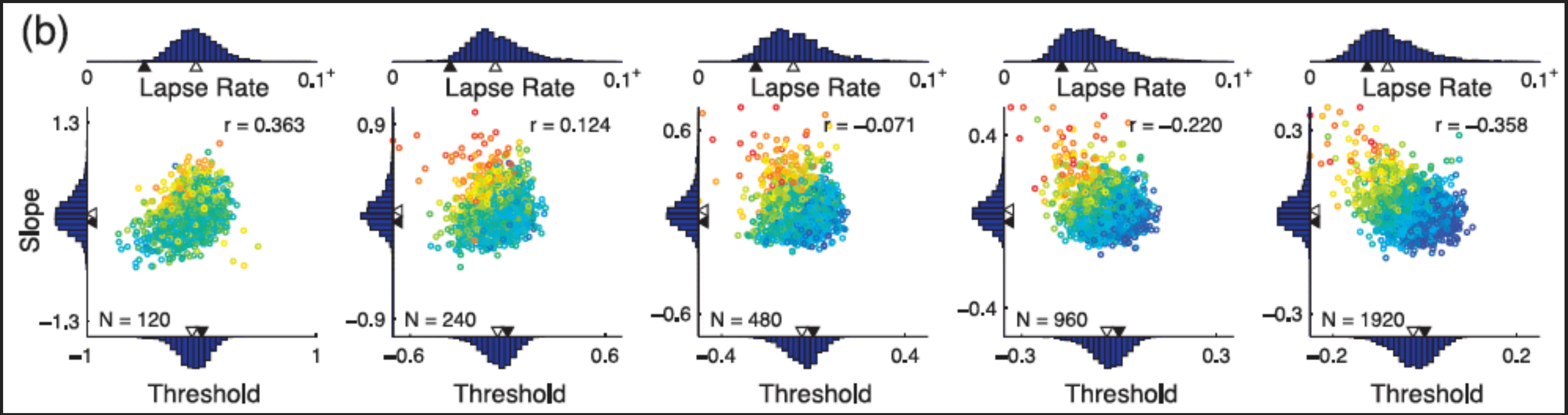


Figure 4. Bayesian parameter estimates for (a) original psi-method assuming a fixed lapse rate, (b) same data refitted with lapse rate free to vary, (c) psi$_{\alpha\beta\lambda}$ (psi^{+}) method, and (d) psi$_{\alpha\beta(\lambda)}$ method. Generating lapse rate was 0.025 for all. Filled triangular symbols indicate generating values of parameters; open triangular symbols indicate the mean of the estimates from all 2,000 simulations. Where only the open triangular symbol is visible, it obscures the filled symbol. Marginal threshold and slope distributions are shown as histograms on axes of the scatterplots. Lapse rate distributions are shown as histograms in separate plot. Note that data here are trimmed in that parameter estimates that exceed the limits of axes are assigned the value of this limit. Note that this was only done for graphical purposes here: Means of parameter estimates reported (open triangular symbols as well as the means and *SE*s presented in Figure 3 [and Figures 6 and 7]) are based on results that were not trimmed.

[from Prins 2013]

- Correlations of **threshold and slopes estimates remain, too** (generating λ = .025; assumed λ = .03)



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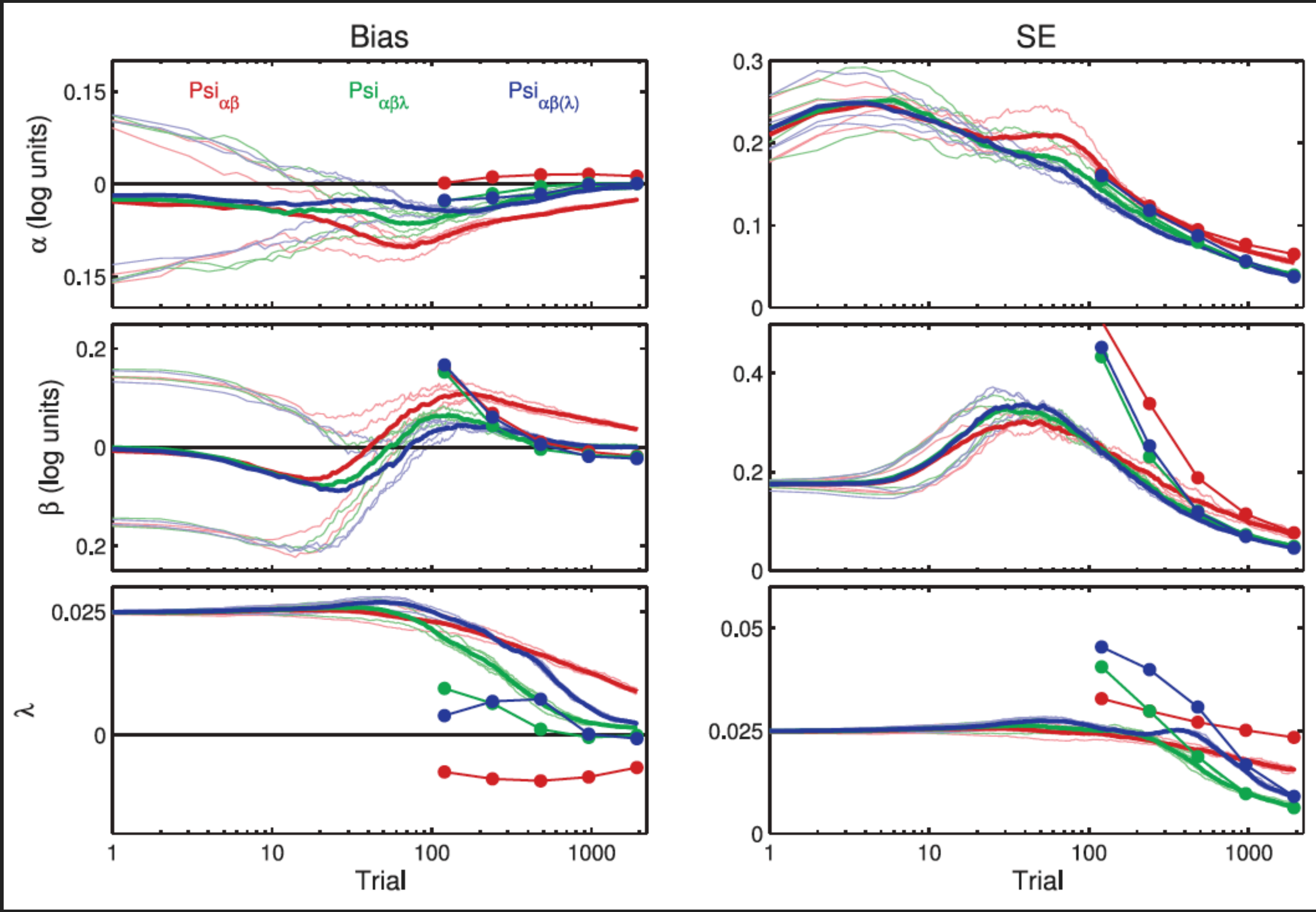
[from Prins 2013]

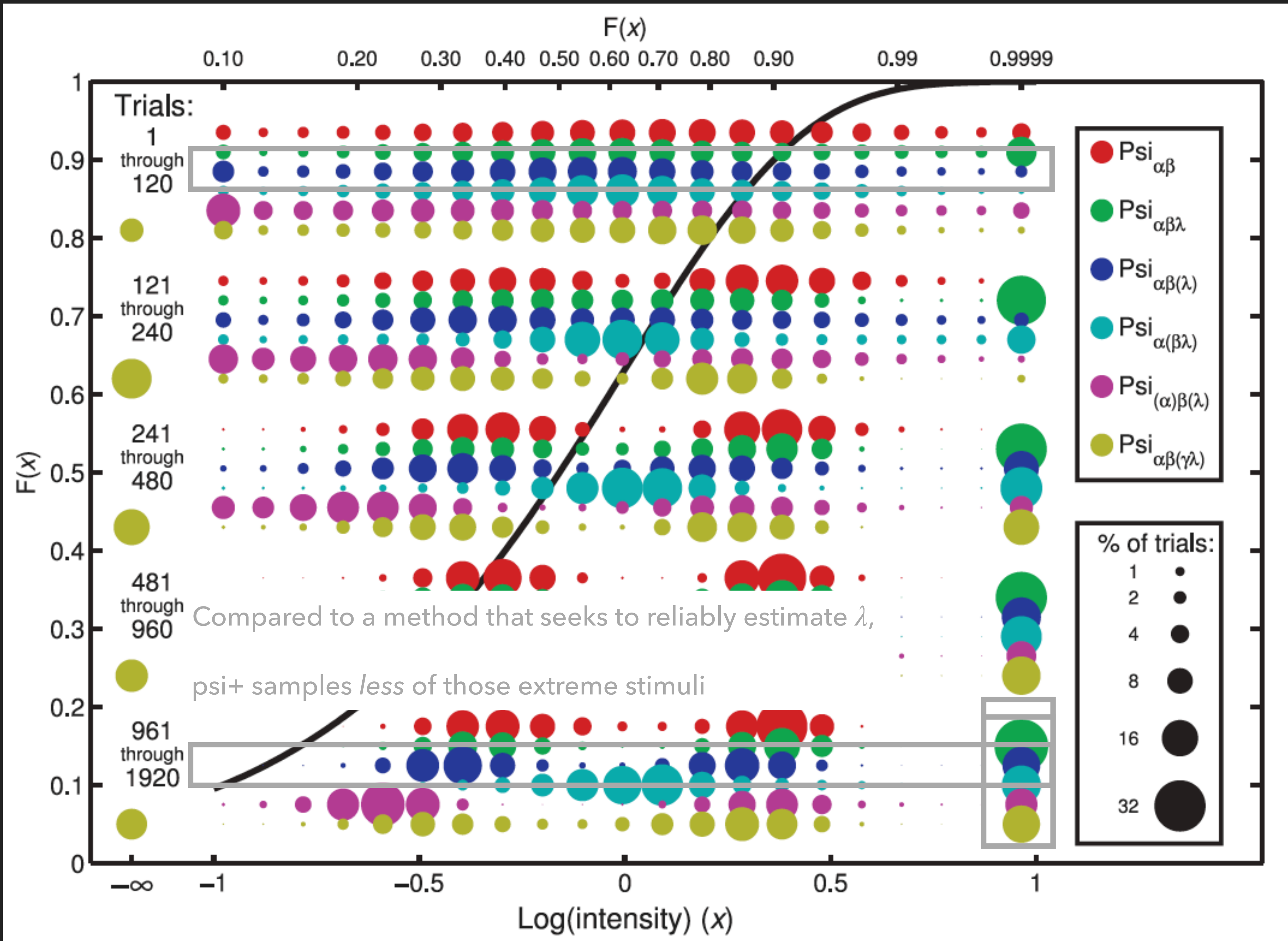
A DIFFERENT ADAPTIVE APPROACH FOR STIMULUS SELECTION

- **Idea behind *psi-marginal adaptive*:** select stimuli so as to *reduce uncertainty about parameters of interest* (e.g., threshold and slope) while taking into account full joint distribution of all parameters (incl. nuisance parameters)

BETTER =)



[from Prins 2013]



Compared to a method that seeks to reliably estimate λ,

psi+ samples *less* of those extreme stimuli

[from Prins 2013]

TIME TO CHANGE OUR ANALYSIS APPROACH?

NOTES ON PRINS (2013)

- The use of a uniform prior seems perhaps ill-advised, compared to a weakly regularizing prior (p. 4).
- Similarly, constraining the lapse rate λ during *estimation* to [0; .1] might not be necessary (p. 7).
- Prins (2013) and similar simulations **assume that the lapse rate does not differ between conditions**. But that's not necessarily true, depending on your design!
- If you're interested in conducting similar analyses, see the tutorial, simulations, and mixed-effects lapsing models at https://github.com/tfjaeger/Tutorial-GLMM. We also have scripts to distribute tasks across the CS cluster.