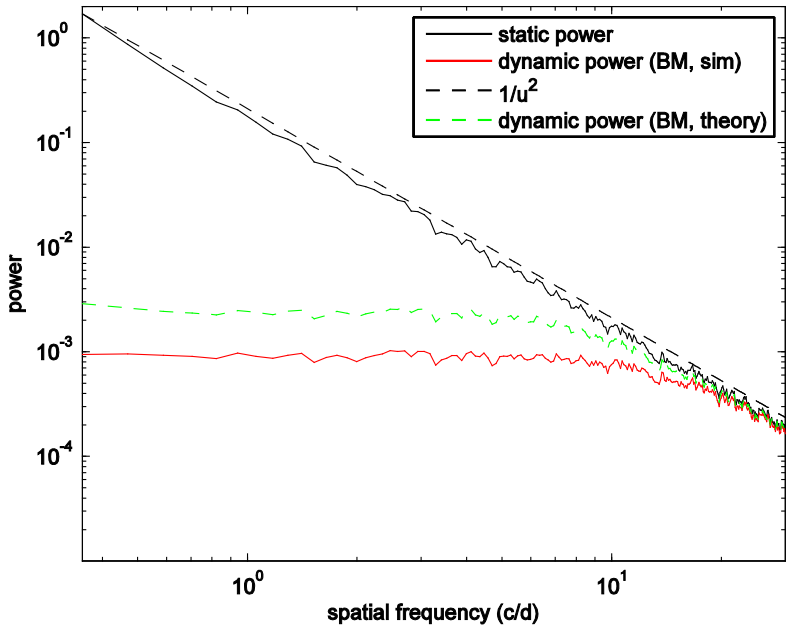
Comparing Welch and JV's theory

4/16/16. Slightly modified from XK file from 7/7/2011.

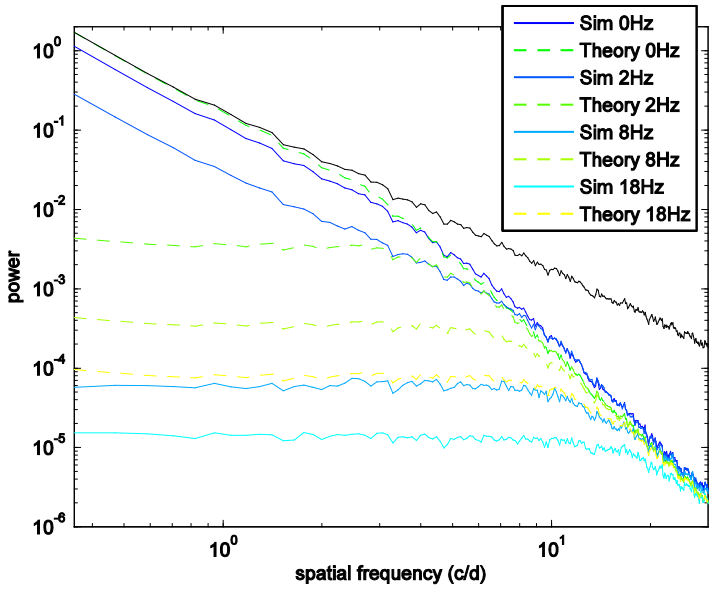
There seems to be some discrepancy between the spectral estimates obtained from Welch and those given by JV's model. Here we focus on BM with a given D to understand the reason for such difference. What follows is from XK notes.

To apply the theory to predict power spectrum of EM, the first step is to estimate the diffusion coefficient of EM. We obtained D=40arcmin^2/s for EM. With this D, we observed that the predicted BM power spectrum has a higher value than that estimated from Welch for EM. This seems to indicate the estimated diffusion constant D is a bit larger than what it should be. However, the program gave accurate estimation of D for simulated random walk, for which we control the value of D. Therefore, there is other cause for this discrepancy.

To understand where the discrepancy comes from, the power spectrum of simulated signals, which are with 1/u^2 power and under simulation of BM with D=40 arcmin^2/s, was estimated under Welch. In this case, the estimated power spectrum should be predicted from our probability theory with BM.



Interestingly, in this simulation, we observed a higher power spectrum under BM theory, compared with the power estimated from Welch. To explore more, we took different temporal slices in the power spectrum .



We can observe from the figure that at 0Hz, the power from BM theory matches well with the estimation from Welch. However, there is a huge discrepancy at 2 Hz. This is due to the leakage of power in Welch estimation. Since the total power across all temporal slices under both BM theory and Welch estimation equals, the leakage at 2Hz for Welch estimation results in lower power at other temporal frequencies (i.e., 6 Hz and 18 Hz in the figure) , compared with what the power should be (the BM theory).

Therefore, when we estimated dynamic power under Welch, we will obtain a power lower than the theoretical predication. And this effect is due to the power leakage in Welch estimation.