Measuring retinal image motion

A point of debate in writing Aytekin at al, 2014 was which reference system should be used to estimate retinal image motion. This is important, as some changes of coordinates may alter retinal velocity.

Consider Fig. 1. Gullstrand's schematic eye models the retinal surface as a sphere with radius R (11.75 mm) and assumes the center of the sphere to be coincident with the eye center of rotation C. It simulates the process of image formation as a two-nodal-points optical system, so that a ray of light going through the first nodal point (N_1) exits the second nodal point (N_2) with a parallel path. The two optical nodal points are located on the line of sight at distances of 5.7 mm (N_1) and 5.4 mm (N_2) from the eye center.

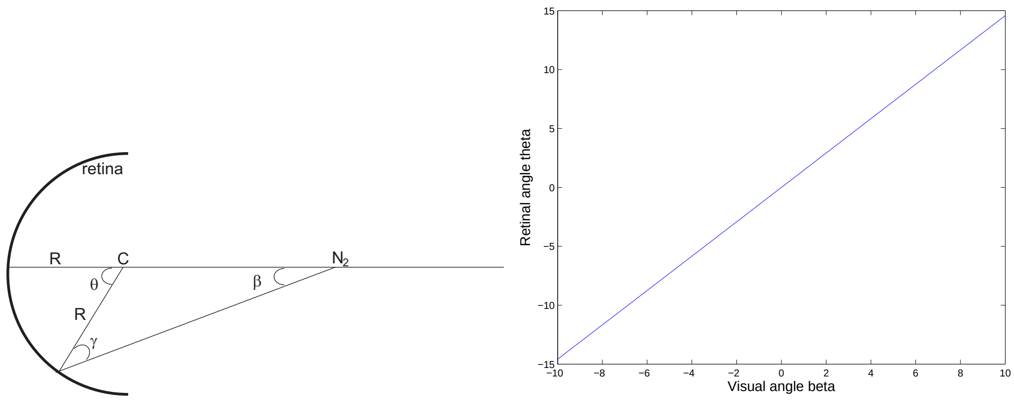

Figure 1: Retinal magnification resulting from using θ rather than the visual angle β for representing retinal image motion. β represents the standard visual angle used in vision science. θ represents the corresponding angle taken from the center of rotation of the retina C.

In this model, it becomes natural to use polar coordinates with a reference system centered at C. This implies use of the angle θ to represent the point of projection on the retina of an object at visual angle β. The two angles θ and β are obviously not coincident: since θ β the retinal image will move at a faster speed (as measured by $\theta(t)$) than the external stimulus (as measured by $\beta(t)$). Use of $\beta(t)$ is in fact not accurate to describe retinal image motion, because it would not give a real angular velocity: C, not N_2 is the center of rotation of the retina in this schematic model.

The relationship between the two angles can be quickly obtained by noticing that:

$$\sin\gamma = \frac{\overrightarrow{CN_2}}{R}\sin\beta$$

Since $\theta = \gamma + \beta$, we obtain:

$$\theta = \beta + \arcsin\left(\frac{\overrightarrow{CN_2}}{R}\sin\beta\right)$$

For very small angles, like those occurring during visual fixation, it is probably sufficient to use the approximation:

$$\theta \approx \beta\left(1 + \frac{\overrightarrow{CN_2}}{R}\right)$$

which, substituting the parameters of Gullstrand's eye model, gives $\theta = 1.46\beta$. Note that the amplification in retinal motion occurs independent of the distance of the fixated target. Thus, for very small movements, if the target moves in space at angular speed $\dot{\beta}$, its angular speed on the retina will be $\dot{\theta} = 1.46\dot{\beta}$

Because of this magnification, the use of a reference frame centered at C may create confusion. Most people are used to work with visual angles in the external field, so, from this point of view, it may seem better to use the angle β when characterizing retinal image motion. However, use of β is (1) essentially equivalent not to use Gullstrand's eye model, and (2) geometrically inaccurate, as this N_2 is not at the center of the retinal sphere. The length of the arc on the retina is larger than what β suggests. This is a real physical change in the motion of the retinal image, not a simple scaling factor caused by a change of coordinates. For a pure rotation around β, the point also changes distance from the retina.

Eye rotation: When the eye rotates by α a very similar geometry applies. Because the nodal point N_2 moves in space, the projection of the point P does not fall in P_1, but moves of an additional angle θ falling onto P_2.

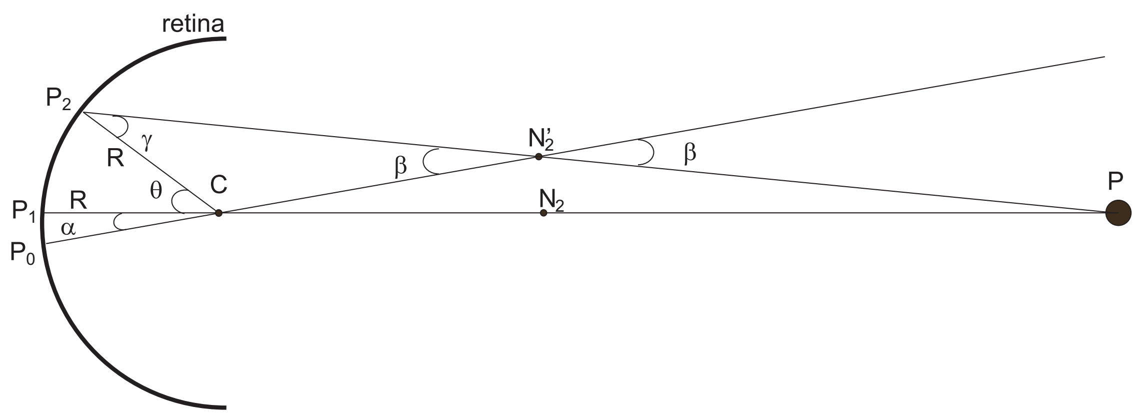

Figure 2: Retinal magnification resulting from using θ rather than the visual angle β for representing retinal image motion. β represents the standard visual angle used in vision science. θ represents the corresponding angle taken from the center of rotation of the retina C.

If the object P is very far away from the eye, we can assume the two projections lines to be parallel. In this case $\alpha = \beta$ and $\gamma = \theta$, so that the angle θ is again given by:

$$\theta = \arcsin(\frac{\overrightarrow{CN_2}}{R} \sin\alpha)$$

Thus for very small rotations, the total angle subtended by the retinal motion will be

$$\alpha + \theta \approx \alpha(1 + \frac{\overrightarrow{CN_2}}{R})$$

and again the stimulus moves on the retina by a factor of 1.46 faster.

Why is this amplification ignored? In an effort to determine why this amplification is ignored in the literature, I went back to Ratliff & Riggs (1950), who discussed this issue and even dedicated a figure to it (Fig. 7). In the caption of the figure, however, there seems to be some misunderstanding.

Ratliff & Riggs (1950) state: “If P is at a considerable distance from the eye, the lines FP and XP may be considered parallel. If these lines are parallel the angles α and β must be equal. Therefore, under these conditions the angle of rotation of the eye and the visual angle subtended by the displacement of the retinal image may be considered equal. ”

But β *is not* the visual angle subtended by the displacement of the retinal image. N_2 is not the center of rotation. The visual angle subtended by the displacement of the retinal image is $\approx \alpha(1 + \frac{\overrightarrow{CN_2}}{R})$.

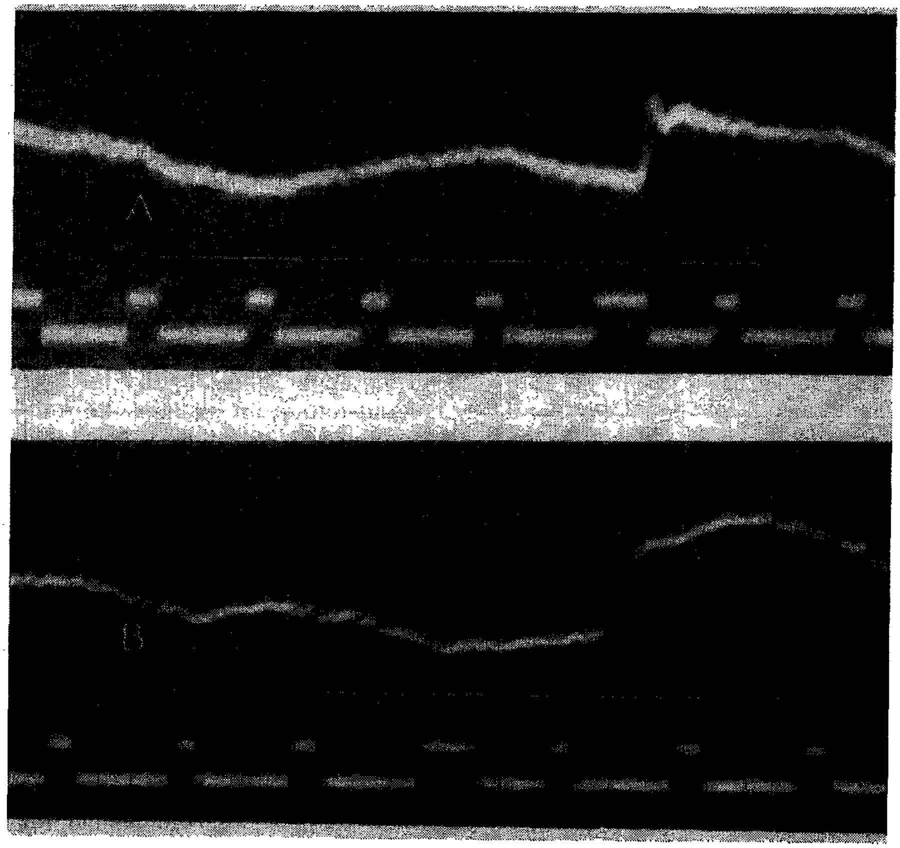

FIG. 6. Control record. Record A obtained with a mirror placed directly on the eye. Record B obtained with mirror mounted on contact lens.

DISCUSSION

Motion of the Retinal Image

On the basis of the records obtained in this experiment it is possible to draw conclusions regarding the motions of the retinal image of an object during fixation. The treatment of the eye as a refracting system may be simplified if it is assumed to have a single principal nodal point. Since this nodal point is not at the center of rotation of the eye, the visual angle subtended by the displacement of the retinal image is not exactly equal to the angle of rotation of the eye. However, Sundberg (15) has shown that if the fixation point is at a considerable distance from the eye, the angle of rotation and the visual angle subtended by the displacement of the retinal image may be considered equal. This point is illustrated in Fig. 7. Assuming these two angles to be equal introduces a slight error when computing the extent of the motion of the retinal image. The error is negligible, however, except for large eye movements occurring when the fixation point is very near the eye.

Movements of retinal images of objects being fixated may be stated in terms of the number of receptors over which a given contour may be ex-

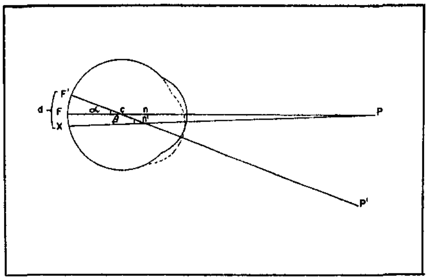

FIG. 7. The effect of the displacement of the nodal point on the position of the retinal image. If the eye turns through the angle Alpha about the center of rotation (c) so that the visual axis moves from P to P′ the part of the retina (F) on which the image of P had previously fallen moves to a new position (F′). However, because the nodal point (n) moves to a new position (n′), the image of P now falls on the retina at the point X. The total displacement (d) of the retinal image of P is the distance which subtends the visual angle Beta. If P is at a considerable distance from the eye, the lines FP and XP may be considered parallel. If these lines are parallel the angles Alpha and Beta must be equal. Therefore, under these conditions the angle of rotation of the eye and the visual angle subtended by the displacement of the retinal image may be considered equal. The following dimensions of the eye were used in all computations: Fc equals 11 mm.; cn equals 5.92 mm.

pected to pass. Polyak (14) describes the dimensions of the receptors as follows:

> The thinnest cones are in the very center of the fovea, where their thickness, in Man, is reduced to almost one mu, corresponding to a visual angle of approximately 12″ to 15″ of arc. The central territory where the cones are almost uniformly thick measures approximately 100 mu across, corresponding to 20′, or one third of a degree, of arc. It contains approximately 50 cones in a line. This area seems to be not exactly circular, but elliptical, with the long axis horizontal, and may contain altogether 2000 cones. This territory, accordingly, represents the peak of the structural perfection of the eye. If one considers that here practically every cone is individually linked with the ganglion cells by means of its own "private" midget bipolar and that accordingly, in this sense it represents an independent functional unit—at least in so far as the "pure cone system" is concerned, it is permissible to conclude that in this central territory the number of visual units corresponds pretty well with the number of cones. In consequence, the size of each of the 2000 receptor-conductor units measures, on the average, 24″ of arc. The size of the units even in this territory varies, however, the centralmost measuring scarcely more than 20″ of arc or even less. Of these—the most reduced cones, and therefore the smallest functional receptor units—there are only a few, perhaps not more than one or two dozen. The size of the units, as given, includes the intervening insulating sheaths separating the adjoining cones from one another.

The amount of angular rotation of the eye during fixation, and hence, the visual angle subtended by the motion of the retinal image, may be roughly summarized as follows: Total movement over a period of three to four sec. is 10 to 20 min. of arc. Total extents of slow movements, drifts, and jerks are approximately five min. of arc. Total extent of small rapid motions: rarely greater than one min. of arc, median value approximately 18 sec.

In terms of the number of receptors across which the retinal image moves, these motions of the eye may be described as follows: over a period of three to four sec. the retinal image may still remain within the limits of the most sensitive area of the fovea. Assuming the average diameter of the cones to be 24 sec., as Polyak has stated, then such movements would have carried the retinal image of the object fixated across a total of 25 to 50 receptors. The slower motions, drifts, and jerks would carry the retinal image of an object fixated across about a dozen receptors.

The small rapid motions are of particular interest, since they appear to represent an irreducible minimum of eye movement such as would characterize optimal fixation on an acuity test object. Since the small rapid motions rarely exceed one min. of arc, movement of the retinal image because of these motions cannot exceed the